Development of a Tool for Risk Based Integrity (RBI) Assessment of Process Components

C9-07

Project duration (Two years with One year extension)

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Summary

The rapid increase of the complexity of oil and gas installations and the escalating demands for their safety and integrity require a risk-based approach for the optimization and management of resources and physical assets. This is of particular importance to the oil and gas industries operating in the Atlantic region, due to the challenge of harsh operating conditions and the uncertainty in the degradation mechanisms to which these installations are subjected to.
This research work was focused on the development of engineering models and tools integrated within a risk-based framework to help answer the following important questions:

- What are the design requirements to ensure the asset integrity?
- What is the status (i.e. integrity) of an asset at any point of time during the operation stage?
- What is the expected remaining life of an asset at any point in time during its operation?
- What data should be collected and how can this data be combined with prior engineering knowledge and experience in order to minimize the required data to be collected and therefore minimize the associated cost?
- When should the next inspection be conducted? What inspection method or tool should be used while maintaining a least cost operation with greatest utility?
- What are the engineering and scheduling requirements for maintenance procedures and the mitigation techniques to be used?
- How effective are the inspection and maintenance in reducing the risk?

The answers to these questions would help the operator in making a well informed decision which will result in more effective asset integrity management and higher level of safety.

1. Introduction

Assets and structures used in industry plants are subjected to several structural deterioration mechanisms during their operation. Structural deterioration may result in damage, deformation, defects or performance degradation. The most common deterioration mechanisms are corrosion, fatigue and creep.

Process components fail while operating even though due diligence has been observed during the design and fabrication stages. Failures of these components pose serious threats to human life, financial investment and the environment. Thus, ensuring the integrity of process components is of paramount importance to its safe operation. The integrity of a component is defined as the ability of the component to perform its required function effectively and efficiently whilst protecting health, safety and the environment (HSE UK, 2009).

A good asset Integrity Management (IM) plan ensures that people, systems, processes and resources required to maintain the asset integrity are in place, in use, and will perform when required over the whole lifecycle of the asset. Furthermore, the IM plan should ensure the prevention of accidents and it should encompass good design, construction and operational practices. Field experience shows that once an offshore process facility is operational, the only way to prevent failure is through frequent inspection and proper maintenance. However, to determine with confidence the extent and interval of necessary inspection and maintenance based on the condition of the component is a challenging task. Maintenance optimization using mathematical models would be a way to reduce the risk of failure of ageing components.

The components of offshore process facilities deteriorate with age. During their life cycle, they will be subjected to many potential damages, such as (Stephens et al., 1995): third party damage; ground movement; material and fabrication defects; and human factors. Past studies indicate that majority of failures are contributed by age-dependent structural degradations (Faber, 2002; Straub, 2004; Khan et al., 2006); hence, the quantification of component integrity can be established by understanding the physics of time-dependent failure processes and its adverse consequences. Traditionally, the codes and standards that are used for regular inspection and maintenance are prescriptive rules based on experience and in
response to significant failure cases. They neither take into account all types of failures, nor the various sources of uncertainty and hence the risk arising from degradation processes. Thus, they result in sub-optimal inspection and maintenance strategies and provide little rationale towards the decision making process.

Proper design procedures should be followed to ensure that all assets are able to withstand the applied loads. Appropriate design approaches should be followed and relevant codes and regulations should be consulted. In traditional design approaches, the integrity of a component is evaluated by comparing the current operating conditions with a design limit state using appropriate safety factors. The actual level of risk is unknown and thus the conservatism built-in the safety factors would lead to irrational cost. In the present risk-based framework, the risk is evaluated and taken as the basis for the design. This is the start point towards ensuring the asset integrity in the developed framework (first part: development of a methodology for risk-based design).

Structural deterioration is a random process as it is a function of random variables such as crack size, location and orientation. The uncertainty and variability of data obtained by inspection sampling of a population suffering structural deterioration are best modeled by stochastic models. In engineering applications, it is often the case that one is forced to make maintenance decisions on the basis of limited data. The probability distribution function estimated using inspection sampling of a random variable such as metal loss or crack size is the likelihood probability of this variable. One can use a likelihood probability to update a known prior distribution using Bayesian updating theory. Handling of the uncertainty and variability of data obtained by inspection sampling to assess the asset integrity is introduced in the last part of the framework (second part: development a model for risk-based integrity modeling).

Even when the asset design is done very carefully, inspection and maintenance are still needed to ensure that the asset will perform its intended function with no threat to the plant integrity during the operation. The asset integrity is established in the design stage.

In the following sections, we describe a framework which will ensure that all assets of an installation will perform their intended functions over its expected operating time and that unexpected failures caused by deterioration mechanisms are kept to a minimum. In the first part of the framework potential risk sources are identified, risk criticality is quantified, and it is explained how risk is incorporated in the design of the process components. Integrity preservation over the operating time is described in the second part. The third part provides a description of the risk-based inspection and maintenance planning process.

2. Background

The first initiatives for the development of risk-based inspection and maintenance modeling were made in connection of welded joints subject to fatigue in fixed steel offshore structures (Skjong, 1985; Madsen et al., 1987; Fujita et al., 1989; Moan et al., 2000). Later, the same methodology was adopted for the design of maintenance strategies for other structures such as tankers (Soares and Garbatov, 1996; Paik et al., 2003); floating, production, storage and off-loading facilities (Lotsberg et al., 1999; Goyet et al., 2002); semi-submersibles; pipelines (Willcocks and Bai, 2000; Desjardins, 2002; Dey and Gupta, 2001), and recently to onshore structures such as process plants (Geary, 2002; Kallen, 2002; Montgomery and Serratella, 2002; Khan et al., 2006), and bridges (Frangopol et al., 2001) and to breakwaters (Noortwijk and Phajim, 1996). The degradation mechanisms such as, fatigue cracking and some aspects of corrosion of steel and concrete structures were considered. Throughout these developments, structural reliability methods have played an important role (Straub, 2004; Faber et al., 2005). Melchers (2006)
introduced an approach for probabilistic corrosion estimation based on the structural reliability theory. Further, Straub and Faber (2006) discussed the computational aspects of risk-based inspection planning for fatigue cracking based on structural reliability theory. The inspection planning for process equipments and marine systems has later evolved from the traditional Quantitative Risk Analysis (QRA) (Khan and Haddara, 2003; Khan et al., 2004; Dey, et al., 2004). A closer review of literature has shown that little information is available on a stochastic risk based methodology for the integrity assessment of offshore process components, in spite of the dangerous threats to integrity of these structures. In order to have a risk predictive capability, the model should be stochastic and dynamic which means that it should be able to model uncertainty and update the degradation processes and failure consequences as new data become available.

3. Objective and outcomes/deliverables achieved

The main objective of this project is to develop an innovative risk-based engineering tool that facilitates asset integrity assessment and maintenance decision making processes. The developed tool should help industry in making well-informed decisions to enhance the safety and integrity of the installation with optimal utilization of physical and financial resources.

To achieve this objective, a risk-based framework for asset integrity assessment was developed. This framework encompasses three main parts:

1- Development of a methodology for risk-based design
2- Development of a methodology for risk-based integrity modeling
3- Development of a methodology for risk-based inspection and maintenance planning

Realistic case studies were used to illustrate the application of the methodologies and the type of results that can be obtained. The results of the application of the developed framework to these case studies are explained in the attached publications.

4. Development of a methodology for risk-based design for asset integrity

It may be noted that codes and standards were developed by classification societies which are currently being used in industry practice to ensure the structural integrity of process components. But this is not a comprehensive approach to see the complete picture of the likelihood of failure of the process system. This is rather a deterministic approach to ensure critical parameters. Consideration of uncertainty of the risk sources in the design is important to ensure the long-term integrity of the process components.

The present work in this section developed a novel methodology for risk-based design of a process component. The scope of the work is comprised of assessing and incorporating different time dependent failure scenarios, which include internal corrosion, external corrosion, stress corrosion cracking, fatigue failure due to start up/shut down and regular pressure fluctuation. It may be noted that time independent failure mechanisms such as third party damage, earth movement, material defect etc. are not considered in the risk based design.

Thus the failure probability obtained for internal and external corrosion and other degradation mechanism will go to the risk based design framework for overall risk estimation. Subsequently all individual component risks will be integrated using fault tree analysis to obtain overall risk of the system.

The unified risk will thus be minimized to individual components by achieving the target safety level of the system.
This research considered uncertainties associated with operational characteristics of process components and included in risk-based design framework. In both internal and external corrosion analysis the study evaluated burst failure probability of process components for the defects created by degradation mechanism. In internal corrosion analysis a relative conservatism scale is discussed for the codes/standards considered in the study. A discussion is provided for code selection for design engineers considering material properties. In some extent the identical procedure is considered for failure probability analysis for defects created by external corrosion. An analytical approach is demonstrated to identify the cause of variability of probability of failure assessed for recommended codes and standards for industry practice.

4.1 Developed risk-based design methodology

The following are the main steps of the developed methodology:

**Step 1:** generate random variables considering distribution and characteristics value with the required confidence level for both load and resistance.

**Step 2:** construct the limit state like \( Z = g(R,S) = R - S \) for load and resistance and calculate probability of failure by using the equations \( \beta = \frac{\mu_Z}{\sigma_Z} \) and \( p_f = 1 - \Phi(\beta) \). The mean and standard deviation will be calculated using either FOSM or AFOSM or SORM method which one is applicable.

**Step 3:** Apply Monte Carlo simulation to calculate probability of failure which is fully calculated on probabilistic approach.

**Step 4:** Calculate risk using the equation, \( Risk = P_f \times C_f \)

**Step 5:** if risk is high, review design parameters of the component and recalculate the risk.

**Step 6:** Up to this, is a component design for a specific mode of failure. Now, likewise, calculate all other modes of failure.

**Step 7:** Integrate the failure modes using FTA with necessary correlation among the failure modes. If there is no correlation, assume independent failure modes. Find a single probability of failure for a process system.

**Step 8:** If risk is not acceptable, find the most detrimental failure modes, and adjust the design parameters of the respective component. Continue the iterative procedure unless you reach the target safety level.

Figure 1 shows the flowchart of the developed methodology.
The process components such as pipeline may be considered for risk based design. The strength of a pipeline deteriorates due to corrosion damage, and generally becomes weaker with increasing age. Hence, the remaining strength of the pipeline is required to be estimated by adopting any suitable method.

**Fig. 1: Risk-based design methodology.**
Different classification societies developed different burst models. It may be noted that the burst pressure models and standards are based on closed end conditions and hoop stress that governs the bursting process. The axial, radial or combined loads such as thermal or bending load are not considered in this study. The notations (burst pressure of defected pipe, yield pressure of defect free intact pipe etc.) are kept the same in this paper as originally developed and presented in the corresponding references. The notations for defect length are specified by \( l \) in some codes and standards (CSA Z662-07, 2007 and Netto et al., 2005) where others (DNV RP-F101, 2004 and ASME B31G, 1995) specify this parameter by \( L \).

4.2 Burst models and standard

4.2.1 CSA Z662-07: For large leaks and ruptures, the limit state function \( g_2 \) for plastic collapse at a surface corrosion defect with total axial length \( (l) \) in mm, and average defect depth \( d_a \) (in mm) are given by CSA Z662-07 standard.

\[
g_2 = r_a - p \tag{1}
\]

where \( r_a \) is the estimated pressure resistance including model error, in MPa

For SMYS (the specified Minimum Yield Strength) > 241 MPa

\[
r_a = e_1 r_c + \left( 1 - e_1 \right) r_o - e_2 \sigma_y \quad (2a)
\]

For SMYS ≤ 241 MPa

\[
r_a = e_3 r_c + \left( 1 - e_3 \right) r_o - e_4 \sigma_y \quad (2b)
\]

\( r_c \) is the calculated pressure resistance, MPa

\[
r_c = r_o \left( \frac{1 - \frac{d_a}{l}}{1 - \frac{d_a}{m \times l}} \right) \quad (3)
\]

\( r_o \) is the pressure resistance for perfect pipe, MPa

For SMYS > 241 MPa

\[
r_o = 1.8 \frac{t \sigma_y}{D}
\]

For SMYS ≤ 241 MPa

\[
r_o = 2.3 \frac{t \sigma_y}{D}
\]

where

\( d_a = d \) is the depth of defect

\( m \) is Folias factor
For $\dot{l}/Dt \leq 50$

$$m = \sqrt{1 + 0.6275 \frac{l^2}{D \times t} - 0.00375 \frac{l^4}{D^3 \times t^2}}$$

For $\dot{l}/Dt > 50$

$$m = 0.032 \frac{l^3}{D \times t} + 3.3$$

e_1$ is a deterministic multiplicative model error term that equals 1.04, $e_2$ is an additive model error term, defined by a normally distributed random variable with a mean of $-0.00056$ and a standard deviation of 0.001469, $e_3$ is a deterministic multiplicative model error term that equals 1.17 and $e_4$ is an additive model error term, defined by a normally distributed random variable with a mean of $-0.007655$ and standard deviation of 0.006506.

4.2.2 DNV RP-F101: In DNV-RP-F101, the capacity for a pipeline with corrosion defect is given by

$$P_{\text{corr}} = 1.05 \frac{2\sigma_u (1 - \gamma_t (d/t))}{(D-t) \left(1 - \gamma_t (d/t) \frac{Q}{Q}ight)} \quad (4)$$

where, $Q = \sqrt{1 + 0.31 \left(\frac{L}{\sqrt{Dt}}\right)^2}$

$L$ is length of defect and $u$ is designed specified minimum tensile strength.

It may be noted that DNV-RP-F101, 2004 proposed two equations for burst estimation: one is to estimate capacity (CP) considering a rectangular defect and the other is to estimate allowable maximum operating pressure (MOP). The MOP equation is not demonstrated here, but is considered in the analysis. In the present analysis, allowable maximum operating pressure (MOP) is referred to as DNV-RP-F101 MOP and capacity pressure is referred to as DNV-RP-F101 CP. In reality, DNV-RP-F101 MOP equation is more conservative than DNV-RP-F101 CP.

4.2.3 ASME B31G: Among the existing criteria for evaluating the residual strength of corroded pipeline, the ASME B31G, 1995 code is still the most widely used criterion. Kiefner et al., 1990a; b recognized that ASME B31G code could be too conservative for some type of defects. They modified the code to develop what is known as the 0.85 $dL$ method. Like the original, the defect length and the defect depth are the only parameters required to define the defect.

The burst pressure defined by ASME B31G, 1995:

$$P^\text{B31G}_b = \frac{2t}{D} (1.1\sigma_f) \left[ \frac{1 - (2/3)(d/t)}{1 - (2/3)(d/t)M^{-3}} \right] \quad (5)$$

where
4.2.4 Netto et al. Model: Netto et al., 2005 developed a burst pressure equation for external corrosion considering the depth of defect, length of defect, width of corrosion, pipeline wall thickness and pipeline diameter. The effect of external corrosion defects was investigated through a series of small-scale experiments and non-linear numerical models based on the finite element method. The experimental and numerical results were then used to calibrate their equation. The burst pressure for defected pipe Eq. 64 was developed with limiting conditions of corrosion defect depth to wall thickness ratio (0.1 \leq d/t \leq 0.8).

\[
\frac{P_{b}}{P_{bi}} = 1 - 0.9435 \left(\frac{d}{t}\right)^{1.6} \left(\frac{L}{D}\right)^{0.4}
\]

where

\[
P_{bi} = \frac{1.1 \sigma_{y} 2t}{D}
\]

4.2.5 Ram pipe requal: The Pipeline Requalification Guidelines Project (Bea and Xu, 1999) developed an equation for burst pressure as

\[
P_{b} = 2.2(t - d) \frac{SMTS}{(D - t) \times SCF}
\]

\[
SCF = 1 + 2 \sqrt{\frac{d}{R}}
\]

where, SMTS is the Specified Minimum Tensile Strength and SCF is the Stress Concentration Factor. The burst equation (Eq. 7) does not consider the corrosion defect length. This may be a significant issue where aspect ratio plays an important role in biaxial stress states.

4.2.6 Kale et al. Model: The Kale et al., 2004 describes a methodology for the predicting the location of internal corrosion damage in gas pipelines considering uncertainties in flow characteristics, pre-existing conditions, corrosion resistance, elevation data, and test measurements. The prediction is then updated using Bayesian techniques based on inspection data. The Kale et al., 2004 model used the three candidate corrosion models. However, in this study only SwRI equation will be considered to study the Kale et al., 2004 model. The core concept of the model is the defect depth (d), whether it exceeds the critical defect depth. The critical defect depth (d_c) was considered as 80% of wall thickness (t) in their model. This is not a burst model but rather a model defining the probability of defect depth exceeding the critical defect depth.

4.3 Sensitivity analysis of burst models

The sensitivity of burst models can be studied by considering any codes and standards. If \( g( ) \) function is considered for sensitivity analysis, the analysis will reveal the parametric affect on the failure function. Gardner et al., 1981 recommend simple correlation coefficients, derived from Monte Carlo simulations, as a reasonable way to rank model parameters. Pearson's product moment correlation coefficient is denoted by \( r \) and is defined as:
4.4 Failure model

The limit state function for corroded pipelines can be written as follows:

\[ g(X) = P_{bdp} - P_{op} \quad (9) \]

where, \( P_{bdp} \) is the burst pressure of the defected pipe, and \( P_{op} \) is the operating pressure. The burst pressure of defected pipe, \( P_{bdp} \), is considered as the resistance and operating pressure, \( P_{op} \), is considered as the load in the limit state function defined by Eq. 9. The burst pressure of defected pipe, \( P_{bdp} \), can be calculated from the respective models as discussed previously. The reliability index \( \beta \) may be obtained from load and resistance variables. Hence, in this case

\[ \beta = \frac{\mu_{P_{bdp}} - \mu_{P_{op}}}{\sqrt{\sigma_{P_{bdp}}^2 + \sigma_{P_{op}}^2}} \quad (10) \]

Using this equation, reliability index, \( \beta \), may be calculated for any code and standard except Kale et al., 2004 model.

The Kale et al., 2004 model does not explicitly define pipe burst limit state function as stated in Eq. 9. For this study the limit state function can be defined as:

\[ g(X) = d_c - d \quad (11) \]

where \( d_c = 80\% \) of wall thickness (t), and \( (d) \) is the defect depth.

Using the limit state equation, Eq. 11 one can now determine reliability index, \( \beta \), for Kale et al., 2004 model considering Eq. 12. Thus

\[ \beta = \frac{\mu_{d_c} - \mu_{d}}{\sqrt{\sigma_{d_c}^2 + \sigma_d^2}} \quad (12) \]

In this study, it was assumed that the defect depth \( (d) \) exceeding critical depth \( (d_c) \) is eventually a failure state.

Once the reliability index, \( \beta \), is calculated for any selected model, the failure probability \( (P_f) \) can be calculated using Eq. 13.

\[ P_f = \Phi(-\beta) = 1 - \Phi(\beta) \quad (13) \]
One can calculate the risk \((R)\) if the consequence \((C_f)\) is known for a specific material and specific location considering Eq. 14.

\[
R = P_f \times C_f
\]  

(14)

4.5 Defect assessment

Corrosion defects may be distributed in the radial, circumferential and axial directions. In general the corrosion defect is defined by a length \((l)\) and through wall thickness depth \((d)\) as this represents the worst case scenario with respect to the applied hoop stress. The defect profile is generally idealized rectangular or parabolic geometric shapes (CSA Z662-07, 2007; DNV RP-F101, 2004; ASME B31G, 1995 and Netto et al., 2005). The subsequent part of the analysis considered internal corrosion defect and external corrosion defect. The analysis and result and discussion of internal corrosion defect is discussed in part I and that of external corrosion defect is discussed in part II.

4.5.1 Internal corrosion defect: Corrosion in the pipeline occurs as individual pits or colonies of pits or in general wall thickness reduction. Figure 2 shows a single, longitudinally oriented, rectangular shaped internal corrosion defect. This type of defect occurs at discrete location and is discontinuous throughout the pipeline length.

![Fig. 2: A simplified internally corroded surface flaw in pipeline](image)

4.5.1.1 Depth of defect \((d)\): Different mathematical models for CO\(_2\) corrosion are used by engineers in the field of oil and gas industry (Nesic, 2007). The NORSOK-model (NORSOK Standard M-506, 1998) was considered by Gartland et al., 2003 for internal corrosion rate estimation. Three candidate corrosion rate models were considered by Kale et al. 2004 including the de Waard-Millams Equation (ASME B31G, 1995) de Waard-Lotz Equation (Waarad and Lotz, 1993) and SwRI (Kale et al., 2004) equation. The first two equations were found to provide comparatively higher corrosion rates with lower correlation with data. Therefore, the third equation, developed by SwRI, is considered for rate estimation.
\[
\frac{da}{dt} = k \times C_I \times 0.0254 \times \left\{ 8.7 + 9.86 \times 10^{-3} (O_2) - 1.48 \times 10^{-7} (O_2)^2 - 1.31 (pH) + 4.93 \times 10^{-2} (pCO_2) (PH_2S) - 4.82 \times 10^{-5} (pCO_2) (O_2) - 2.37 \times 10^{-3} (pH_2S) (O_2) - 1.11 \times 10^{-3} (O_2) (pH) \right\} 
\]

\text{mm/year} \quad (15)

\[C_I = 1 - e^{-\frac{d \cdot L}{L_o}}\]  \quad (16)

In Eq. 15, \(pCO_2\) is partial pressure of carbon dioxide in the mixture, \(pH_2S\) is the partial pressure of hydrogen sulfide in the mixture, \(O_2\) is the concentration of oxygen in parts per million, \(k\) is the modeling error, and \(C_I\) is the inhibitor correction factor given by Eq. 16. Since the inhibitor’s effect diminishes with the pipeline length \((L)\), the inhibitor correction factor \((C_I)\) uses an exponential model along the pipeline length. In Eq. 16, \(A\) is the model parameter, \(L\) is the pipeline length and \(L_o\) is the characteristic length (hence \(L_o = 1000 \text{ km}\)) to describe the effect of the inhibitor. The inhibitor effect is considered at the inlet and no other inhibitor injector was considered throughout the remaining pipeline length. Table 1 presents random variables with probabilistic data responsible for internal corrosion in a demonstrative pipeline scenario. The depth of defect \((d)\) was calculated considering corrosion rate, \(da/dt\), times \(T\), 20 years design life in this study.

Table 1: Probabilistic data for the random variable- depth of defect \((d)\) (wet gas pipeline corrosion growth parameters) (partly (Waard and Lotz, 1993))

<table>
<thead>
<tr>
<th>Variables</th>
<th>%CO_2 (mole)</th>
<th>O_2 (ppm)</th>
<th>pH</th>
<th>%H_2S (mole)</th>
<th>K, corrosion model error</th>
<th>A, Inhibitor factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Lognormal</td>
</tr>
<tr>
<td>(\mu)</td>
<td>4</td>
<td>4800</td>
<td>5.5</td>
<td>0.05</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>COV</td>
<td>0.25</td>
<td>0.30</td>
<td>0.18</td>
<td>0.08</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

4.5.1.2 Length of defect: The defect depth \((d)\) for internal corrosion was estimated considering model parameters i.e, \%pCO_2, \%pH_2S etc. But this kind of model, which considers model parameters i.e, \%pCO_2, \%pH_2S etc. is not available for defect length \((l)\) estimation. In fact corrosion measurements revealed that there is no correlation between the depth of defect \((d)\) and length of defect \((l)\), instead its observed that for a given depth of defect \((d)\) there is a range of associated length of defect \((l)\) (Amirat et al. 2009). For example, for a depth of 20% wall thickness, the length \((l)\) varied from 8 to 608 mm. However, Zimmerman et al., 1998 suggested corrosion defect length \((l)\) can be assumed by Weibull distribution with COV of 0.50. The COV 0.50 means shape parameter \((\beta')\) of Weibull is 2.1. The scale parameter \((\theta)\) was calculated considering Eq. 17. The calculation considered cumulative distribution \(F(l) = 0.90\) and characteristic length \((l_c)\) as 80% of the diameter of the pipeline. The mean defect length thus evaluated is 340 mm, which represents the defect length after the design life, \(T = 20\) years. Table 2 shows the probabilistic data for the defect length \((l)\).

\[
P(l \geq l_c) = 1 - F(l) = e^{-\left(\frac{l}{\theta}\right)^{\beta'}} = \int_{l_c}^{\infty} f(l) dl
\]

\text{(17)}
Table 2: Probabilistic data for the random variable: length of defect ($l$)

<table>
<thead>
<tr>
<th>Variables</th>
<th>$l$-defect length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Weibull</td>
</tr>
<tr>
<td>$\mu$</td>
<td>340</td>
</tr>
<tr>
<td>COV</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Table 3: Probabilistic models of the basic variables for material- API 5L X 65.

<table>
<thead>
<tr>
<th>Variables</th>
<th>$\sigma_u$/MPa</th>
<th>$\sigma_y$/MPa</th>
<th>D/mm</th>
<th>t/mm</th>
<th>$P_{op}$/MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Normal</td>
<td>Normal</td>
<td>Gumbel</td>
</tr>
<tr>
<td>$\mu$</td>
<td>531</td>
<td>448</td>
<td>713</td>
<td>20.24</td>
<td>17.12</td>
</tr>
<tr>
<td>COV</td>
<td>0.05</td>
<td>0.07</td>
<td>0.001</td>
<td>0.001</td>
<td>0.08</td>
</tr>
</tbody>
</table>

4.5.2 Results and discussion: The probability of failure of different candidate models was calculated and assessed. A pipeline length of 1000 km was considered in the analysis with a single inhibitor injector at the inlet. The material considered API 5L X65 for design. The analysis used First Order Second Moment (FOSM) and Monte Carlo (MC) simulation methods considering the variables given in Table 4. The results are presented in Fig. 3 for FOSM method.

Table 4: Results obtained for different codes/standards.

<table>
<thead>
<tr>
<th>Codes/Standard</th>
<th>FOSM</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_f$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Kale et al., 2004</td>
<td>$1.4*10^{-28}$</td>
<td>-</td>
</tr>
<tr>
<td>DNV RP-F101, 2004</td>
<td>$3.1*10^{-6}$</td>
<td>4.51</td>
</tr>
<tr>
<td>Netto et al., 2005</td>
<td>$1.4*10^{-4}$</td>
<td>3.63</td>
</tr>
<tr>
<td>ASME B31G, 1995</td>
<td>$4.8*10^{-4}$</td>
<td>3.29</td>
</tr>
<tr>
<td>CSA Z662-07, 2007</td>
<td>$4.2*10^{-3}$</td>
<td>2.63</td>
</tr>
<tr>
<td>DNV RP-F101, 2004 MOP</td>
<td>$2.7*10^{-2}$</td>
<td>1.92</td>
</tr>
<tr>
<td>RAM PIPE (Bea and Xu, 1999)</td>
<td>$3.4*10^{-2}$</td>
<td>1.82</td>
</tr>
</tbody>
</table>
Fig. 3: Failure probability $P_f$ for different standards and models using burst and critical depth in the limit state equation a) normal graph b) logarithmic graph excluding Kale et al., 2004.

The simulated results by FOSM and Monte Carlo have been compared and found they closely match with one another at the tail end of the pipeline length. The results were presented in Table 4.

Examination of the failure probability ($P_f$) calculated by using different candidate models, as illustrated in Figure 3 and summarized Table 4, indicates a gap or discrepancy. RAM PIPE (Bea and Xu, 1999) and DNV RP-F101, 2004 MOP calculate greater failure probability than other models including CSA Z662-07, 2007; ASME B31G, 1995; Netto et al., 2005 and Kale et al., 2004. It may be noted that in
Figure 6, DNV RP-F101 MOP refers to the failure probability for allowable maximum operating pressure equation, whereas DNV RP-F101 CP refers to failure probability for capacity pressure equation. There is significant difference in the estimation of probability of failure: DNV RP-F101 CP calculates in the range of $10^{-6}$, whereas DNV RP-F101 MOP calculates in the range of $10^{-2}$. It may be noted that DNV RP-F101 MOP equation is not a capacity equation as that of DNV RP-F101 CP equation, rather maximum allowable operating pressure equation.

The failure probability estimate for ASME B31G and Netto et al., 2005 models are consistent and exhibit limited relative variation along the pipeline length. Netto et al., 2005 concluded that ASME B31G and DNV RP-F101 are conservative models, which is consistent with the findings of this study. The analysis of Netto et al., 2005 indicated DNV RP-F101 MOP failure probability in Figure 3, which considered maximum operating pressure. It could be more rational if they compared their model with the DNV RP-F101 CP burst capacity equation. However, in Fig 16 of their study they compared the result obtained from Netto et al., 2005, ASME B31G and DNV RP-F101 MOP with experimental result. If the present probabilistic study is compared with the Netto et al., 2005 study, it can be safely said that the codes/models that calculate greater failure probability compared to Netto et al., 2005 are over conservative.

![Relative position of the codes/standards in ‘Conservatism Scale’ considering remaining strength (burst pressure) and operating pressure.](image)

**Fig. 4:** Relative position of the codes/standards in ‘Conservatism Scale’ considering remaining strength (burst pressure) and operating pressure.

A relative ranking of conservatism in the candidate models is illustrated in Figure 4. In Figure 4, let $H$ denotes the load (operating pressure), $F$ and $C$ denote bursting pressure of RAM PIPE (Bea and Xu, 1999) and Netto et al., 2005 model, respectively. The remaining strength or bursting pressure calculated by RAM PIPE (Bea and Xu, 1999) model is closest to the load, that’s why its failure probability is highest. As $C$ is far away from $H$ that’s why Netto et al., 2005 calculates less failure probability. The other codes/standards in Figure 4 lie in between $C$ and $F$ except DNV RP-F101 CP. To define over conservative or under conservative the position of $A$ is important which denotes the experimental remaining strength. According to Netto et al., 2005 model the position of $C$ is below the position of $A$ the position denotes the true remaining strength, evaluated by experimental study. Again, the results of
present study (Table 4) suggest that the location point of DNV RP-F101 CP, B, in Figure 4, must be located closest to A since DNV RP-F101 CP calculates least failure probability among the burst models. Therefore capacity equation developed by DNV RP-F101 CP can be considered as the best estimator of the remaining strength. Finally, it can be stated that the codes and standards which calculate less remaining strength than actual (experimental) remaining strength (line A) can be assumed as over conservative. The relative position presented in Fig 4 is also supported by the experimental data studied by Freire et al., 2006. In their analysis they have demonstrated that ASME B31 is over conservative compared to DNV-RP-F101 CP. A deterministic analysis of remaining strength (burst pressure) for different codes and standards in Figure 5, shows that the relative conservatism scale remains true for 0.15<d/t<0.42.

![Graph showing remaining strength calculation](image)

**Fig. 5:** A deterministic approach of remaining strength calculation shows that the conservatism scale remains true for 0.15<d/t<0.42.

The conservatism of failure or burst pressure estimated by the codes depends on geometry of the pipe, geometry of the defect, and the material. Past research concentrated on the behavior of sharp defects (machined V-shaped notches and slits), but subsequently the work was extended to consider artificial and real corrosion defects.

Many failure criteria like ASME B31G, DNV-RP-F101, CSA Z662-07 etc. are originally based on flow stress dependent failure criteria of the NG-18 equations (Kiefner et al., 1973) and have been assumed as plastic collapse failure criteria. In many tests, failure was preceded by significant amounts of ductile tearing and some of the steels had low toughness. The geometry term was empirical and the flow stress was adjusted to fit the test results. This leads to empirical definition of the flow stress which is conservative, since biased towards the behavior of older steels (Cosham et al., 2007). The NG-18 (Kiefner et al., 1973) equations were developed from tests of V-shaped notches, not blunt, part-wall defects. Therefore, many methods for assessing the corrosion based on the NG-18 (Kiefner et al., 1973) equations have a conservative bias when applied to tests of blunt, part-wall defects.

Efforts have been made to develop in the accuracy of failure criteria by better describing the effects of reference stress and geometry. DNV-RP-F101 has used finite element analyses of blunt, part-wall
defects to determine the form of the geometry, and has considered the form of the reference stress in more detail. The failure criteria have been validated against burst tests of modern line pipe steels containing blunt, part-wall defects or real corrosion defects. Modern line pipe steels have a higher toughness than older steels, such that the failure of blunt part-wall defects is controlled by plastic collapse (where plastic collapse is defined in terms of the ultimate tensile strength), and hence the scope of toughness can be better accounted in burst models.

The Kale et al., 2004 model is not a failure model, rather a model which calculates probability of defect depth ($d$) exceeding the critical depth, ($d_c$). In this study it was assumed that defect depth represents a failure state when it exceeds critical depth. There is a limitation in this assumption, but for simplicity it is nonetheless assumed in this analysis. In their analysis Kale et al., 2004 assumed the critical depth is 80% of wall thickness for oil and gas pipelines. The other features in Kale et al., 2004 model is that, this model considers only depth of defect ($d$), and does not consider the length of defect ($l$). The length of defect ($l$) has due importance on the failure probability of gas pipelines. Another feature in the limit state equation of the Kale et al., 2004 model is that the resistance (critical depth ($d_c$)) is constant, and the load (depth of defect ($d$)) is variable; the opposite observation is noticed in other models/standards.

For more details of internal corrosion defect assessment, see Hassan et al., 2011 appended to this report.

4.6.1 External Corrosion: External corrosion is an oxidation process on external surfaces which are normally exposed to subsea water. It is a two step reaction process in which the loss of metal and production of electrons occur at the anodic area, and the consumption of these electrons occurs at the cathodic area. Hence, the overall external corrosion rate is dictated by the ratio of the anodic area to the cathodic area, the concentration of the cathodic reactant and, to a lesser extent, the resistivity of the local environment, which determines the volume of ion transformation between the anode and cathode (Palmer and King, 2008). For the corrosion to continue, the electrons remaining on the metal surface must be removed by a cathodic reaction. Typical cathodic reactions are hydrogen evolution and oxygen reduction. The higher the availability of oxygen to the metal surface, the higher the potential rate of corrosion. Oxygen access to the bare metal surface increases as the temperature of the water decreases, or as the flow rate over the surface increases. The splash zone of offshore process components, like risers, is, therefore, at the highest risk of corrosion in cold water moving at a high velocity (Palmer and King, 2008).

4.6.2.1 Depth of defect: The reliability analysis of corroded pipeline requires the probabilistic specification of the basic variables $d$ and $l$, which define the corrosion defect. A basic corrosion profile of external corrosion defect is given in Figure 6. It may be noted that there is no correlation between the depth of defect ($d$) and length of defect ($l$), instead it is observed that for a given depth of defect ($d$), there is a range of associated length of defect ($l$) (Amirat et al., 2009). For example, for a depth of 20% wall thickness, the length varied ($l$) from 8 to 608 mm. However, depth of defect ($d$) and length of defect ($l$) can be assumed to be Weibull distributed with COV of 0.50 suggested by Zimmerman et al., 1998. Thus the probability of defect depth ($d$) greater or equal than its characteristic value ($d_c$) is expressed by:

$$P(d \geq d_c) = 1 - F(d) = e^{-\left(\frac{d}{d'_c}\right)^\beta} = \int_{d_c}^{\infty} f(d)dd$$

where $F(d)$ is the cumulative distribution of defect depth ($d$), ($\beta'$) is the shape parameter and ($\theta$) is the scale parameter of the distribution and $dd$ is small change in defect depth.
4.6.2.2 Length of defect: In the same way the probability of defect length \( l \) greater or equal than its characteristic value \( l_c \) is expressed by

\[
P(l \geq l_c ) = 1 - F(l) = e^{-\left(\frac{l}{\theta}\right)^{\beta'}} = \int_{l_c}^{\infty} f(l)\,dl
\]  

(19)

The COV 0.50 means shape parameter \((\beta')\) of the Weibull distribution for defect depth \( d \) and defect length \( l \) in equations 18 and 19 both are 2.1. The scale parameter \((\theta)\) for defect depth \( d \) and defect length \( l \) were calculated using equations 18 and 19. The calculation considered cumulative distribution \( F(d)=F(l)=0.90 \) and characteristic depth \( (d_c) \) as 1.5\% of the thickness \( (t) \) of the pipeline for each year. Similarly, the characteristic length \( (l_c) \) was considered as 4\% of the diameter \( (D) \) of the pipeline for each year. The mean defect depth \( (d) \) is thus 0.1807 mm/yr and the defect length \( (l) \) is 17 mm/yr. The study also assumed the linear addition of defect depth \( (d) \) and defect length \( (l) \) for each year. These reflect that after the design life of \( T=20 \) years, the mean defect depth \( (d) \) and mean defect length \( (l) \) would be 3.61 mm and 340 mm, respectively. Table 5 shows the probabilistic data for the defect depth \( (d) \) and defect length \( (l) \).

Table 5: Probabilistic data for the random variables- depth of defect \( (d) \) and length of defect \( (l) \).

<table>
<thead>
<tr>
<th>Variables</th>
<th>( d )-defect depth (mm/yr)</th>
<th>( l )-defect length (mm/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Weibull</td>
<td>Weibull</td>
</tr>
<tr>
<td>M</td>
<td>0.1807</td>
<td>17</td>
</tr>
<tr>
<td>COV</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Fig. 6: A simplified internally corroded surface flaw in pipeline
Table 6: Probabilistic models of the basic variables for material- API 5L X 65.

<table>
<thead>
<tr>
<th>Variables</th>
<th>$\sigma_u$/MPa</th>
<th>$\sigma_y$/MPa</th>
<th>$D$/mm</th>
<th>$t$/mm</th>
<th>$P_{op}$/MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Lognormal</td>
<td>Lognormal</td>
<td>Normal</td>
<td>Normal</td>
<td>Gumbel</td>
</tr>
<tr>
<td>$\mu$</td>
<td>530</td>
<td>447</td>
<td>713</td>
<td>20.24</td>
<td>17.12</td>
</tr>
<tr>
<td>COV</td>
<td>0.05</td>
<td>0.07</td>
<td>0.001</td>
<td>0.001</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 7: Probability of failure ($P_f$) obtained for different codes/standards at the end of the design life, $T=20$ yrs.

<table>
<thead>
<tr>
<th>Codes/Standard</th>
<th>FOSM</th>
<th>Monte Carlo</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_f$</td>
<td>$\beta$</td>
</tr>
<tr>
<td>DNV RP-F101 CP</td>
<td>5.3*10^{-5}</td>
<td>3.87</td>
</tr>
<tr>
<td>Netto et al. 2008</td>
<td>2.4*10^{-4}</td>
<td>3.48</td>
</tr>
<tr>
<td>ASME B31G</td>
<td>5.3*10^{-4}</td>
<td>3.27</td>
</tr>
<tr>
<td>CSA Z662-07</td>
<td>9.9*10^{-3}</td>
<td>2.33</td>
</tr>
<tr>
<td>DNV RP-F101 MOP</td>
<td>3.6*10^{-2}</td>
<td>1.80</td>
</tr>
<tr>
<td>Bea et al., 1999</td>
<td>5.5*10^{-2}</td>
<td>1.59</td>
</tr>
</tbody>
</table>

4.6.3 Result and Discussion: The failure probability of different burst models recommended by codes/standard was calculated and compared for external corrosion analysis considering data of Table 5 and Table 6. The analysis used First Order Second Moment (FOSM) and Monte Carlo (MC) simulation method and considered 20 years design life. The results obtained by two solution approaches are closely matching as noticed in Table 7. This expresses the accuracy of the results. The analytic (FOSM) result of the analysis is presented in Figure 7. Analyzing Figure 7, a significant difference is noticed in the value of $P_f$ for different burst models recommended by codes/standards and the models developed by individuals. The highest value of probability of failure 5.5×10^{-2} is observed for Bea et al., 1999 model while the lowest probability of failure 5.3×10^{-5} is observed for DNV RP-F101 CP model.
Fig. 7: Failure probability $P_f$ for different codes/standards using burst in the limit state equation $a$) normal graph and $b$) logarithmic graph

The cause of variability may be answered by sensitivity analysis of the failure functions $g(\ )$ for different codes/standards.
Table 8: Probabilistic models of dimensionless parameters.

<table>
<thead>
<tr>
<th>Dimensionless Parameter</th>
<th>Type</th>
<th>( W = \frac{d}{t} )</th>
<th>( X = \frac{D}{t} )</th>
<th>( Y = \frac{\sigma}{\sigma_y} \text{ or } \sigma_s )</th>
<th>( Z = \frac{l}{D} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \mu )</td>
<td>Weibull</td>
<td>0.1792</td>
<td>35.22</td>
<td>0.6794 (Netto et al., ASME B31G)</td>
<td>0.4717</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.5730 (CSA)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.5558 (DNV)</td>
<td></td>
</tr>
<tr>
<td>COV</td>
<td>Normal</td>
<td>0.5050</td>
<td>0.0014</td>
<td>0.1130 (Netto et al., ASME B31G)</td>
<td>0.5030</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.1157 (CSA)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.1123 (DNV)</td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 8:** Graphical representation of sensitivity analysis of dimensionless parameter by Monte Carlo method.
The sensitivity is analyzed in terms of dimensionless parameters \{W=d/t, X=D/t, Y=\sigma_h/(\sigma_y \text{ or } \sigma_u) \text{ and } Z=l/D\} of the failure functions \(g()\) using the Monte Carlo approach. The analysis considered the data given in Table 8 for dimensionless parameters in the \(g()\) function. The analysis considered the remaining parameters (other than dimensionless) constant in the failure function. The results are presented in Figure 8. The sensitivities for the dimensionless parameter \(X=D/t\) are observed to be insignificant for each codes/standards. This suggests that either a change in \(D\) or \(t\) has little effect on the failure \(g()\) function. It may be noted that the failure \(g()\) function for Netto et al model, does not truly contain any \(D/t\) ratio, whereas the other codes/standards contain \(D/t\) ratio in the stress concentration factor, \(M\) or \(Q\).

The other dimensionless parameters \{\(W=d/t, X=D/t, Y=\sigma_h/(\sigma_y \text{ or } \sigma_u) \text{ and } Z=l/D\}\) are observed to be sensitive in each of the codes/standards. It means a small change in \((W, Y, \text{ or } Z)\) has a significant effect on the failure function \(g()\). Dimensionless parameter \(Y\) has the most significant effect on the failure function \(g()\), followed by \(W\) and \(Z\). It may be noted that the failure \(g()\) function for Netto et al model, does not truly contain any \(D/t\) ratio, whereas the other codes/standards contain \(D/t\) ratio in the stress concentration factor, \(M\) or \(Q\).

### Table 9: Importance factor in the reduction factor \(P_{bi}\).

<table>
<thead>
<tr>
<th>Importance factor (in % contribution) in reduction factor, (P_{bi})</th>
<th>Netto et al model</th>
<th>ASME B31G</th>
<th>CSA Z662</th>
<th>DNV RP F101</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d)</td>
<td>76</td>
<td>64</td>
<td>63</td>
<td>59</td>
</tr>
<tr>
<td>(l)</td>
<td>21</td>
<td>34</td>
<td>35</td>
<td>41</td>
</tr>
<tr>
<td>Other parameters ((D &amp; t))</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Considering equation 20, the burst pressure of the defected pipe is a function of intact pressure and reduction factor.

\[
P_{dp} = f\left\{\text{intact burst, 'reduction factor' } P_{bi}\right\} \\
= f\left\{\text{intact burst } f(F,t,\sigma_y,D) \text{'reduction factor' } f(d,t,l,D)\right\}
\]

(20)

Although it is observed that parameters in the intact burst pressure are not responsible for variation in remaining strength estimation of corroded pipe for different codes and standards, a further sensitivity analysis of reduction factor, \(P_{bi}\), reveals the cause of variability in remaining strength. Figure 9 shows the sensitivity analysis of \(P_{bi}\) factor. According to Figure 9, the contribution of \(d\) and \(l\) in the burst model equation is responsible for variation in calculated remaining strength. Table 9 further illustrates the
importance of the dimensionless parameters in the failure function. The contribution of the parameter $d$ in the reduction factor $P_{bi}$ is found the most significant, which is followed by $l$. According to Table 10, the basic variable $d$ contributes 76% in Netto et al. model while the contribution is 59% in DNV-RP-F101 model. Similar observation was noticed for the basic variable $l$, in which the contribution is 21% in Netto et al. model while the contribution is 59% in DNV-RP-F101 model. The insignificant contribution is again observed for $D$ and $t$.

![Graphical representation of sensitivity analysis of $P_{bi}$ factor by Monte Carlo method.](image)

Fig. 9: Graphical representation of sensitivity analysis of $P_{bi}$ factor by Monte Carlo method.

In the present study, a significant variation is observed in probability of failure ($P_f$) for different burst models recommended by codes/standards and also by individuals. The variation in probability of failure is due to differences in remaining strength estimation by different models. The cause of variability in remaining strength is contributed by $d$ and $l$ in the burst models recommended by codes/standards.

5. Development of a methodology for risk-based integrity modeling (RBIM)

The risk-based integrity modeling provides a framework to quantify the risks posed by aging components, based on structural degradation processes. The operating life risk may be used as criteria for decision making regarding the inspection and maintenance. The integrity refers to the soundness of the component to perform its desired functions. The major threats to asset integrity in process components have been identified at first. These are age-based degradation processes, such as corrosion and cracking. Based on detailed literature review, the critical structural degradation processes threatening the integrity of process components, such as pipes, bends and tees are identified as Uniform Corrosion (UC), Pitting
Corrosion (PC), Erosion Corrosion (EC), Stress Corrosion Cracking (SCC), Corrosion Fatigue Cracking (CFC) and Hydrogen Induced Cracking (HIC). Thus, the essential steps of the risk based integrity modeling are the estimation of probability of these degradation failures and their consequences should they fail. The overall framework for the risk based integrity modeling has been presented in Figure 10. The probability of failure is estimated using Bayesian prior-posterior analysis of potential degradation processes. The consequence analysis estimates the consequence of an undesirable event occurrence in terms of cost of failure, inspection and maintenance, damage to human life, and environment. The consequences of failure are expressed in terms of cost (in dollars) associated with failure, inspection and maintenance. The annual equivalent cost (AEC) of failure consequence is combined with cumulative density function (CDF) of failure probability to estimate the operational life risk profile may be estimated as given below:

$$R(j) = F[p(\theta / y, j)] \times AEC(j)$$

(21)

where, $R(j)$ is the risk of failure due to a degradation (in dollar) in the $j^{th}$ interval, $F[p(\theta / y, j)]$ is the CDF of posterior probability of failure and $AEC(j)$ is the annual equivalent cost, corresponding to the inspection and maintenance interval, $j$. The AEC may be computed from the equivalent rate costs of failure, inspection and maintenance. Thus, finding the optimal inspection and maintenance interval reduces to finding a value of maintenance interval that minimizes the operational risk. At the optimal risk point, the risk will be reduced to as low as reasonably practicable (ALARP) level and at the same time the maintenance interval is maximized, thus avoiding unwanted maintenance and its associated costs. The risk in dollar is compared with the company’s operating budget (as risk acceptance criteria) to make a decision on maintenance. The risk acceptance criteria typically relate to the safety of personnel and risk to environment. They will be reflected in the corporate’s annual operating and maintenance budgets. By plotting the operational risk curve over maintenance intervals, the optimum interval may be obtained as the period corresponding to the minimum risk.

An engineering replacement analysis is used to obtain an optimal replacement strategy. The same formula as in Eq. 21, with $j$ being the replacement interval may be used. The annual equivalent cost (AEC) is computed as a summation of the annual equivalent of failure recovery, inspection and maintenance costs. The annual equivalent of failure recovery cost may be estimated using the annuity factor, indicating a series of future payments towards the failure cost for a specified number of years. The expected cost of inspection and maintenance involves periodic payments that increase by a constant amount from period to period, as a function of the age of component. This increasing trend may be modeled using arithmetic gradient (Park, 2007). Then, the AEC is combined with CDF of posterior probability of failure to estimate the operational life risk and economic service life of components. The optimum replacement interval will be obtained from the economic service life analysis of components. The methodology presented in Figure 10 consists of four parts; identification of potential degradation mechanisms, stochastic degradation modeling for estimating the likelihood of failure, economic consequence analysis for estimating the consequences of failure, optimization of maintenance strategy and, testing and validation using actual field data.
Fig. 10. Risk-based integrity methodology.
5.1 Threats to the Integrity of Process Component

Leaks and rupture are the principal causes of hydrocarbon release, fire, and explosions in process facilities (API 581, 2000). Studies indicate that corrosion is the principal cause of about 15% of leakage occurrences (HSE UK, 2002). In nine and a half years, 44.70% of the mechanical failures leading to hydrocarbon releases from offshore facilities in the UK resulted from corrosion or other related degradations (HSR UK, 2002). The direct annual cost of corrosion in the USA is assessed to be 276 billion USD, which represents 3.1% of the GNP, while about 121 billion USD is spent on corrosion control (Koch et al., 2000). The direct cost of corrosion in industrialized countries in billions of USD is reported (Bhaskaran et al., 2005): Japan (59.02), Russia (55.01), Germany (49.26), UK (8.51), Australia (7.32) and Canada (3.38). These figures show that age-based corrosion and related cracking degradation is an economic problem, which needs to be addressed on a priority basis. In Canada, the environmentally induced defects, such as metal corrosion, stress corrosion cracking and hydrogen induced cracking were responsible for 40% of the natural gas pipelines failures and 38% of hazardous liquid releases (Stephens et al., 1995). Further, it is reported that corrosion accounts for 21% of failures in submarine gas pipelines, and erosion-corrosion modes account for 24.6% of pipe leakages in process plants (Googan and Ashworth, 1990). Moreover, 40% of the accidental hydrocarbon releases to the environment are corrosion related. Therefore, the investigation and mitigation of corrosion and cracking and its effects are one of the main actions required to reduce the frequency of hydrocarbon releases, to maximize the production, and to improve the safety of offshore process operations. Better inspection and maintenance optimization strategies need a reliable determination of degradation mechanisms and their consequences. This can be achieved with risk analysis by combining the stochastic degradation modeling with economic consequence analysis (Faber, 2002). Different methods are required for the inspection and maintenance of different degradation processes. Kowaka (1994), Melchers (2001), Goyet et al., (2002), and Khan and Howard (2007) reported that the main threats to the integrity of process facilities are several types of corrosion (Figure 11). Further, Kallen (2002), Straub (2004), and Straub and Faber (2005) have reported that the major degradation mechanisms threatening the integrity of structural components consist of various types of cracks (Figure 12).

Corrosion is the loss of material as a result of a chemical reaction between a metal and its environment. Based on literature study (Stephens et al., 1995; Kallen, 2002; Khan et al., 2006), the critical structural degradation mechanisms threatening the integrity of assets are UC, localized PC and EC, cracking, such as SCC, CFC, and HIC. Uniform corrosion is defined as the uniform or regular removal of metals from the surface (Jones, 1996). For uniform corrosion, the corrosive environment must have the same access to all parts of the metal surface, and the metal itself must be uniform in terms of metallurgy and composition. Uniform corrosion results in the thinning of wall thickness until the wall is penetrated leading to leaks or breakdown of equipment (Mansfeld, 1987). The localized attack of corrosive environment on an otherwise resistant surface produces pitting corrosion (Jones, 1996). The combination of the corrosive fluid and high flow velocity results in erosion corrosion. A stagnant or slow flowing fluid will cause a low or modest corrosion rate, but the rapid movement of the corrosive fluid physically erodes and removes the protective corrosion product film, exposing the reactive metal beneath, thus accelerating corrosion. Sand or suspended slurries enhances erosion and accelerates erosion corrosion attack on metal. The attack follows the directions of localized flow and turbulence around surface irregularities.
The brittle fracture of a normally ductile alloy, in presence of a corrosive environment or cyclic loading is known as cracking (Jones, 1996). The amount of cracking per unit time either in length or depth is expressed in terms of cracking rate. The stress corrosion cracking is the cracking induced by the combined influence of static tensile stress and a corrosive environment. The required tensile stresses may be in the form of directly applied stresses or in the form of residual stresses. The process in which a metal fractures prematurely under conditions of simultaneous corrosion and repeated cyclic loading at lower stress levels or fewer cycles is known as corrosion fatigue cracking. Hydrogen induced cracking means the severe loss of ductility caused by the presence of atomic hydrogen in the metal lattice (Jones, 1996). Hydrogen absorption may occur during electroplating, welding, pickling, cathodic protection or other processes that favor the production of nascent hydrogen at the surface.

Fig. 11: Material degradations- various types of corrosion.
Bayes Theorem

Structural degradation modeling is often viewed as an iterative process of integrating, accumulating and interpreting information capturing the physics of failure process. The analysts can assess the current state of knowledge regarding the degradation level, gather new integrity data to infer the question of future degradation, and then update and refine the current understanding to incorporate new data. Bayesian inference provides a logical and quantitative framework for this. Bayesian approach to degradation modeling starts with the formulation of a model that is expected to describe the degradation process. The prior distributions of unknown parameters of the model may then be formulated, which are meant to capture the beliefs about the degradation before actually viewing the data (evidence). After observing data, the Bayes theorem may be applied to obtain the posterior distributions for those unknowns, which takes account of both the prior and system data. From these posterior distributions, predictive distributions for future observations of corrosion and cracking may be computed.

Probability is a degree of belief, that is, how much one thinks that something is true based on the evidence at hand. In the face of uncertainty in degradations, one can make the best inference based on the inspection data and any prior knowledge that one might have, reserving the right to revise the present knowledge if new information comes to light. Bayes theorem encapsulates this process of learning as more data become available. It states how to update the prior probability distribution, $p(\theta)$, with a likelihood function, $p(y/\theta)$, mathematically, to obtain the posterior distribution as:

Fig. 12: Material degradations—various types of cracking.
The posterior density \( p(\theta / y) \) summarizes the total information, after viewing the data and provides a basis for inference regarding the parameter, \( \theta \). Denominator of (Eq. 22), i.e., \( \int p(\theta)p(y/\theta)d\theta \) is known as the normalizing factor. The application of the Bayesian methods in risk analysis is limited due to the challenge of computing normalizing factors.

5.2.1 Conjugate Pair Distributions: The conjugate pairs are those distributions, whose posterior can be directly obtained from the prior and likelihood parameters and hence no computations are needed. For example, the Gamma prior and likelihood provides a Gamma posterior with a combination of the prior and likelihood parameters. The natural conjugate pairs for exponential families are presented in Table 10. The use of conjugate pair makes it simple to carry out the process of Bayesian updating. However, in some cases the concept of conjugate pairs does not yield realistic posteriors. Some literature conveniently assumes there are conjugate pairs for degradation process, for easy computation of posteriors, which is not the case in real life. This introduces significant uncertainty in the analysis. Distributions like, Weibull, lognormal and extreme value, do not lend themselves easily to the Bayesian updating. Other alternatives are the use of simulation methods or numerical approximations to determine the posterior distributions (Robert and Casella, 1999). In this project, simulation methods, such as Metropolis-Hastings (M-H) algorithm are used for the posterior development. To compare the results of simulation methods, analytical approximation, such as Laplace Approximation (LA) method are used.

<table>
<thead>
<tr>
<th>Prior Distribution</th>
<th>Likelihood</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \pi(\theta) )</td>
<td>( f(y/\theta) )</td>
<td>( \pi(\theta / y) )</td>
</tr>
<tr>
<td>Normal ( N(\mu,\tau^2) )</td>
<td>Normal ( N(\theta,\sigma^2) )</td>
<td>( N(\rho(\sigma^2 + \tau^2 \rho), \rho \sigma^2 \tau^2) )</td>
</tr>
<tr>
<td>Gamma ( G(\alpha,\beta) )</td>
<td>Poisson ( P(\theta) )</td>
<td>( G(\alpha + y, \beta + 1) )</td>
</tr>
<tr>
<td>Gamma ( G(\alpha,\beta) )</td>
<td>Gamma ( G(\nu,\theta) )</td>
<td>( G(\alpha + \nu, \beta + y) )</td>
</tr>
<tr>
<td>Beta ( Be(\alpha,\beta) )</td>
<td>Binomial ( B(n,\theta) )</td>
<td>( Be(\alpha + y, \beta + n - y) )</td>
</tr>
<tr>
<td>Beta ( Be(\alpha,\beta) )</td>
<td>Negative Binomial ( Neg(m,\theta) )</td>
<td>( Be(\alpha + m, \beta + y) )</td>
</tr>
</tbody>
</table>
The different probabilistic models which can be used to describe major corrosion degradation mechanisms will be discussed in this section. The distributions of corrosion samples can be established in several ways; including frequency diagrams, plotting data using probability graphs, and conducting the goodness of fit tests for the distributions (Halder and Mahadevan, 2000). The parameters of the distribution can be estimated using the methods of least squares, moments and maximum likelihood estimates.

In order to develop the best suitable prior probability models for different corrosion and cracking mechanisms, several distributions have been tested using data extracted from the literature. For this purpose, the uniform corrosion data has been extracted from Anghel and Lazar (2005), Melchers (2003), Lawson (2005), McLaughlan and Stuetz (2004) and Paik et al. (2003). For pitting corrosion, the data has been extracted from Melchers (2005), Scarf and Laycock (1996), Laycock et al. (1990) and Sankaran et al. (2001). For erosion corrosion, the data has been extracted from Vinod et al. (2003), Melchers (2006), Salama (2000) and Abdusalam and Stanley (1993). Similarly, the SCC data has been extracted from Lu et al. (2003), Engelhardt et al (2003), Michael et al. (2004) and Shibata et al. (2007), the CFC data from Robert et al. (2005), Sankaran et al. (2001), Chuang et al (1998) and Bolotin (2001), and the HIC data from Krom et al. (1997), Erik (2004), Dell (1973) and Woodtli et al. (2000) has been used for probabilistic testing.

The extracted data has been tested with standard probability distributions, like Normal, Lognormal, 3P-Lognormal, Weibull, 3P-Weibull, Exponential, 2P-Exponential, Type1 Extreme Value, Gamma and Beta using the statistical software Minitab and developed subroutines in Matlab. The goodness of fit test has then been performed using the adjusted A-D statistic and the best fit is reported as the one with smallest Anderson-Darling (A-D) statistic. The more relevant prior distributions with A-D statistic for probability plot, maximum likelihood estimates and the method of least squares are reported. Please refer to Thodi et al., (2009) for additional information on prior development. The sample probability plots are presented for uniform corrosion in Figure 13, the pitting corrosion in Figure 14, and the erosion corrosion in Figure 15.

Two Matlab subroutines, Prohrfit and Crackfit have been developed for testing the candidate distributions using maximum likelihood estimates. The log-likelihood statistic has been used to compare the goodness of fits and to estimate the parameters of the distributions. The parameters, such as, location and scale parameters are estimated using 95% confidence intervals. The interactive distribution fit tool, dfittool has been used to prepare the probability plots of the data and to estimate the log-likelihood parameter. The probability plots have been developed using the least square estimate (LSXY) also. The A-D test statistic and correlation coefficient (CC) statistic have been used for comparing the goodness of fit. The lower value of A-D statistic and higher value of CC statistic suggested better fit. The mean, standard error, 95% of upper and lower bounds of the probability have also been computed. A summary

<table>
<thead>
<tr>
<th>Dirichlet</th>
<th>Multinomial</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(\alpha_1, ..., \alpha_k)$</td>
<td>$M_k(\theta_1, ..., \theta_k)$</td>
<td>$N(\mu, 1/\theta)$</td>
</tr>
<tr>
<td>Gamma</td>
<td>Normal</td>
<td>$G(\alpha + 0.5, \beta + (\mu - y)^2 / 2)$</td>
</tr>
<tr>
<td>$Ga(\alpha, \beta)$</td>
<td>$N(\mu, 1/\theta)$</td>
<td>$G(\alpha + 0.5, \beta + (\mu - y)^2 / 2)$</td>
</tr>
</tbody>
</table>

5.3 Prior probability modeling
of more relevant and less relevant models for UC, PC, EC, SCC, CFC and HIC prior selection is reported in Table 11.

Table 11: Summary of best models for probabilistic corrosion/cracking degradations.

<table>
<thead>
<tr>
<th>Degradation Mechanism</th>
<th>Best Fitting Model</th>
<th>2nd Best Fitting Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform Corrosion</td>
<td>3P-Weibull</td>
<td>3P-Lognormal</td>
</tr>
<tr>
<td>Pitting Corrosion</td>
<td>Type 1 Extreme Value</td>
<td>3P-Weibull</td>
</tr>
<tr>
<td>Erosion Corrosion</td>
<td>Weibull</td>
<td>3P-Lognormal</td>
</tr>
<tr>
<td>Stress Corrosion Cracking</td>
<td>Type 1 Extreme Value</td>
<td>3P-Weibull</td>
</tr>
<tr>
<td>Corrosion Fatigue Cracking</td>
<td>Lognormal</td>
<td>3P-Lognormal</td>
</tr>
<tr>
<td>Hydrogen Induced Cracking</td>
<td>3P-Weibull</td>
<td>3P-Lognormal</td>
</tr>
</tbody>
</table>

5.4 Likelihood probability modeling

The inspection data obtained from an offshore production facility operating in the North Sea has been used to validate the selected priors of corrosion degradations. The data used for uniform corrosion is the data obtained for the Gas Condensate (GC) system. This data is used to obtain the distribution for uniform corrosion as the data is observed to follow a uniform wall loss. It was observed that data obtained for the Gas Export (GE) system, in the above mentioned facility, follows the localized or pitting corrosion. The data associated with HP Drilling Mud (HP) system has been observed to follow the erosion pattern. The data includes the minimum and average inspection readings acquired during the period 1997 to 2003. The nominal diameter of its components varied from 19.05 to 508 mm. For precise estimation of corrosion rates, inspection data has been divided into several groups, namely, straight pipes and features. Features include bends, tees, reducers, flanges and valves (Khan and Howard, 2007). Three major components: straight pipes, bends and tees were only considered in the analysis.
Fig. 13: Sample probability plots for uniform corrosion, data from Anghel and Lazar (2005).
Fig. 14: Sample probability plots for pitting corrosion, data from Scarf and Laycock (1996).
Fig. 15: Sample probability plots for erosion corrosion, data from Melchers (2006).
5.4.1 Subsystem Description: The flow lines of GC consist of lines from high pressure compressor K1301 to Cooler E1303 of nominal wall thickness varying from 5.54 to 17.48 mm. In the system GE, the subsystem 6 isometrics is included in Figure 16. The subsystem 6 essentially consists of gas export lines from K3201B to first stage after cooler (0.75, 1.0, 1.5, 6 and 8 inch lines), K3201C to after coolers, K3201 A/B (0.75, 1.0, 1.5 and 6 inch lines), K3201A, first stage compressor (3 and 6 inch lines), and K-3201A to after/inter coolers (6, 8 inch lines). The nominal wall thickness of its components varied from 3.91 to 23.01 mm. The flow lines in HP Drilling Mud system consist of high pressure mud lines of module 2 and 16, with wall thickness of components varying from 5.49 to 19.05 mm.

5.4.2 Analysis Methodology: The statistical analysis task has been divided into two: the precise estimation of corrosion rates and the testing of corrosion rates with standard probability distributions. The method outlined in Khan and Howard (2007) has been used to compute the corrosion. The collected data is first analyzed to identify uniform or localized degradation. In the case of uniform degradation, time dependent regression analysis and in the case of localized degradation, the extreme value analysis has been carried out for estimating the rates of degradation. In the regression analysis, regressor variable considered is the period of exposure (T) of each system and the response variable is the loss of wall thickness (Y) over such duration. The inspected data (NDT) is then regressed to get the degradation rate, k which is represented by the slope of the regression line, Y = kT + C, where C is referred as the wall thickness loss (C = 0) at the start of service, i.e., at (T = 0).

Corrosion rates for localized material degradation were estimated using an extreme value model (Khan and Howard, 2007; Melchers, 2005). If different set of samples is obtained through inspection, one can select the extreme values from each sample set and then construct a distribution in the extreme value analysis. The extreme value equations are summarized in Table 12. For more details, please refer to Thodi et al. (2009).
Fig. 16: Sample subsystem 6 (of gas export lines) isometric drawing.

Table 12: Extreme value distributions.

<table>
<thead>
<tr>
<th>Maximum value</th>
<th>Minimum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(y) = \frac{1}{\alpha} \exp[-y - \exp(-y)] )</td>
<td>( f^{-1}(y) = \frac{1}{\alpha} \exp[y - \exp(y)] )</td>
</tr>
<tr>
<td>( F(y) = \exp[-\exp(-y)] )</td>
<td>( F^{-1}(y) = 1 - \exp[-\exp(y)] )</td>
</tr>
<tr>
<td>Where ( y = \frac{x - \lambda}{\alpha}; \alpha &gt; 0 )</td>
<td>Where ( y = \frac{x - \lambda}{\alpha}; \alpha &gt; 0 )</td>
</tr>
</tbody>
</table>

where, \( x \) is wall loss or pit depth, \( \lambda \) is location parameter, and \( \alpha \) is scale parameter.

The annual wall losses were plotted using the simple regression method for uniform corrosion and the extreme value distribution for localized and erosion corrosion data. The extreme value probability
plot, which is obtained by plotting the ordered wall loss versus the cumulative probability, i.e., \((-\ln(-\ln(f(wall~loss))))\) for each year is developed. The observation of a good linear fit, suggests the appropriateness of choosing extreme value distributions for such data. Such plots can then be used to estimate the location and scale parameters, mean, median and most likely wall losses and the yearly wall loss corresponding to 95% confidence intervals. The cumulative exposure times and the corresponding wall loss values for the 95% confidence interval are used for the estimation of corrosion rates. The predicted wall losses corresponding to the confidence intervals of 0.95 over several inspection years may then be plotted against the cumulative exposure times to estimate the actual corrosion rate either by linear or power law model. The estimated corrosion rate data has been tested with probability distribution models like, Normal, Lognormal, 3P-Lognormal, Weibull, 3P-Weibull, Exponential, 2P-Exponential and Type 1 Extreme Value using Minitab. The goodness of fit test has been performed using the A-D statistics and the best fit is reported as the one with smallest A-D statistic. The tested models with their A-D test statistics are summarized in Table 13.

<table>
<thead>
<tr>
<th>Type of corrosion</th>
<th>Systems or component</th>
<th>Most relevant distributions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Best fitting model</td>
</tr>
<tr>
<td>Uniform corrosion</td>
<td>Straight pipes</td>
<td>3P-Weibull</td>
</tr>
<tr>
<td>Pitting corrosion</td>
<td>Feature-bends</td>
<td>Type 1 Extreme Value</td>
</tr>
<tr>
<td>Erosion corrosion</td>
<td>Feature-tees</td>
<td>3P-Weibull</td>
</tr>
</tbody>
</table>

No such data were available in the case of different cracking as the industry policy is to replace the cracked component immediately rather than repairing it.

5.5 Posterior probability modeling

There are four methods for computing the posterior probability models using the known prior and likelihood models. They include (Ghosh et al., 2006): analytical approximations, such as numerical integration techniques and Laplace approximations; data augmentation methods, such as E-M (Expectation-Maximization) algorithm; Monte Carlo direct sampling and MCMC (Markov Chain Monte Carlo) methods, such as M-H algorithm and Gibb’s sampling. If the problem under consideration does not involve a conjugate prior-likelihood pair, the posterior estimation cannot be performed in closed form; analytical or Monte Carlo methods are needed (Tierney and Kadane, 1986). In the present study, the developed prior and likelihood for degradations, such as Weibull, Lognormal (with two and three parameters) and Type 1 Extreme Value do not lend themselves easily to Bayesian updating. The main problem is that there is no distribution class on the parameters that is preserved under Bayesian updating (Bedford and Cooke, 2001). This means that simulation methods are the best ways to determine the posterior distributions of such combinations. The use of M-H algorithm in conjunction with a particular choice of prior has been suggested (Bedford and Cooke, 2001; Robert and Casella, 1999). In the present project, the M-H algorithm has been implemented. In order to compare the results of the M-H algorithm, an analytical Laplace approximation method has also been used. By comparing the results of both the
estimations against the values obtained from known conjugate pairs, the best suitable posterior development method has been concluded.

5.5.1 Metropolis – Hastings (M-H) algorithm: The M-H algorithm is a rejection-sampling based algorithm used to generate a sequence of samples following a probability distribution that is difficult to sample directly (Metropolis et al., 1953; Hastings, 1970). This sequence is used in MCMC simulations to approximate a distribution or to compute an integral. In Bayesian applications, the normalizing factor is often extremely difficult to compute, so the ability to generate the posterior samples without actually knowing this constant of proportionality is a major virtue of this algorithm (Berg, 2004). The algorithm generates a Markov chain in which each state \( x^{t+1} \) depends only on the previous sample state \( x^t \). The algorithm uses a proposal density \( q(x', x^t) \), which depends on the current state \( x^t \), to generate the new proposed sample \( x' \). The proposal is accepted as the next value \( (x^{t+1} = x') \) if \( \alpha(x', x^t) \) drawn from a uniform distribution, \( U(0,1) \) if:

\[
\alpha(x', x^t) = \frac{p(x')q(x^t, x')}{p(x^t)q(x^t, x')}
\]

(23)

If the proposal is not accepted, then the current value of \( x \) is retained; i.e., \( x^{t+1} = x^t \). The proposal density may be a multivariate Gaussian distribution centered around the current state \( x^t \);

\[
q(x', x^t) \sim N(x^t, \sigma^2)
\]

where, \( q(x^t, x') \) is the probability density function for \( x^t \) given the previous value \( x^t \). This proposed density would generate samples centered around the current state with variance, \( \sigma^2 \). The acceptance of such generated samples will be based on Eq. 23. Theoretical background of the M-H algorithm has been summarized in the next section.

5.5.1.1 Theory behind the M-H algorithm: A proposal density \( q(x', x^t) \) is assumed, where \( \int q(x', x^t)dx^t = 1 \). It is assumed that the density is to be depending only on the current state of process, since dealing with Markov chains. This value is to be interpreted as saying that when a process is at the point \( x^t \), the density generates a value \( x' \) from \( q(x', x^t) \). For this to happen, \( q(x', x^t) \) should satisfy reversibility condition (Chib and Greenberg, 1995). But mostly, it will not; one might find for example, that for some \( (x', x^t) \):

\[
p(x^t)q(x', x^t) > p(x^t)q(x^t, x')
\]

(24)

In this case, the process moves from \( x^t \) to \( x' \) too often and from \( x' \) to \( x^t \) too rarely. A convenient way to correct this condition is to reduce the number of moves from \( x^t \) to \( x' \) by introducing a probability \( \alpha(x^t, x') < 1 \), that the move is made. The \( \alpha(x^t, x') \) is known as the probability of move. If the move is not made, the process again returns \( x^t \) as a value from the target distribution. Thus, the transition from \( x^t \) to \( x' \) are made according to \( p_{MH}(x', x^t) = q(x', x^t)\alpha(x', x^t) \), \( x' \neq x^t \), where the probability of move, \( \alpha(x^t, x') \) is yet to be determined. From Eq. 24, it is obvious that the movement from \( x' \) to \( x^t \) is
not made often. One should therefore define \( \alpha(x', x^t) \) to be as large as possible and, since it is a probability its upper limit is 1. But now, the probability of move \( \alpha(x', x^t) \) is determined by requiring that \( p_{MH}(x', x^t) \) satisfies the reversibility condition, because then (Chib and Greenberg, 1995):

\[
p(x^t)q(x^t, x^t)\alpha(x^t, x^t) = p(x^t)q(x^t, x^t)\alpha(x^t, x^t) = p(x^t)q(x^t, x^t)
\]

Therefore, \( \alpha(x^t, x^t) = \frac{p(x^t)q(x^t, x^t)}{p(x^t)q(x^t, x^t)} \) \( \quad \text{(26)} \)

where, \( \alpha(x^t, x^t) \) is set as 1 (the upper limit). If the inequality in Eq. 25 is reversed, we set \( \alpha(x^t, x^t) = 1 \), and derive the \( \alpha(x^t, x^t) \) as above. The probabilities \( \alpha(x^t, x^t) \) and \( \alpha(x^t, x^t) \) are introduced to ensure that the two sides of Eq. 25 are in balance or, in other words, \( p_{MH}(x', x^t) \) satisfies the reversibility. Thus, in order for \( p_{MH}(x', x^t) \) to be reversible, the probability of move must be set to:

\[
\alpha(x^t, x^t) = \min \left[ \frac{p(x^t)q(x^t, x^t)}{p(x^t)q(x^t, x^t)} \right] , p(x^t)q(x^t, x^t) > 0 \\
= 1 \quad \text{otherwise.} \quad \text{(27)}
\]

The M-H algorithm is specified by its proposal density, \( q(x^t, x^t) \). If a candidate value is rejected, the current value is taken as the next item in the sampling sequence. The calculation of \( \alpha(x^t, x^t) \) does not require the knowledge of normalizing constant of \( p(.) \) because it appears both in numerator and denominator. If the proposal density is symmetric, i.e., \( q(x^t, x^t) = q(x^t, x^t) \), then the probability of move \( \alpha(x^t, x^t) \) reduces to \( p(x^t) / p(x^t) \), hence, if \( p(x^t) > p(x^t) \), the chain moves to \( x^t \); otherwise it moves with probability given by \( p(x^t) / p(x^t) \). In this project, the M-H algorithm has been implemented in Matlab software. The algorithm implementation details can be obtained from elsewhere (Makowski and Wallach, 2007; Makowski et al., 2002; Robert and Casella, 1999; Chib and Greenberg, 1995; Tierney, 1994).

5.5.2 Laplace approximation: Laplace method (Laplace, 1986) is used for approximating the parameters of the posterior densities that is useful in Bayesian applications when direct estimations are difficult. The Laplace approximation is very handy tool when a normal approximation posterior is reasonable and can be especially useful in higher dimensions when other methods fail (Gill, 2002). The basic idea is to carry out a Taylor series expansion around the maximum likelihood estimate value (i.e., mode), ignore the negligible terms, and normalize. The derivation of the approximation in one dimension is simple and it starts with a posterior density of interest calculated by the likelihood times the specified prior:

\[
p(\theta | y) \text{ is proportional to } p(\theta)L(y | \theta) \quad \text{(28)}
\]
where, \( p(\theta) \) is the prior, \( L(y/\theta) \) is the conditional likelihood function and, \( p(\theta/y) \) is the posterior. It is assumed that this distributional form is nonnegative, integrable, and single peaked about the distribution mode \( \hat{\theta} \). The standard reference for approximating the Bayesian posteriors with Laplace method (Tierney and Kadane, 1986). Further, it was showed that how the Laplace approximation can be a handy tool for calculating the parameters of the Bayesian posteriors (Ghosh et al., 2006; Tanner, 1996; Kass, 1993; Tierney et al., 1989 a and b; Tierney and Kadane, 1986).

### 5.5.2.1 Theory behind the Laplace approximation:

A computable approximation for the posterior mean and variance of a smooth function of the parameter that is nonzero on the interior of the parameter space is introduced (Tierney and Kadane, 1986). Let \( g(\theta) \) be a smooth, positive function on the parameter space, with a maximum at \( \hat{\theta} \). The posterior mean of \( g(\theta) \) can be written as:

\[
\mu = \mathbb{E}[g(\theta)/y] = \frac{\int g(\theta)e^{-nh(\theta)}d\theta}{\int e^{-nh(\theta)}d\theta}
\]

where, \( e^{-nh(\theta)} = p(\theta)L(y/\theta) \). It is a common practice to approximate the denominator integral by an approximating normal curve centered at the posterior mode and having variance equal to minus the inverse of the second derivative of the log posterior density at its mode. It will produce reasonable results as long as the posterior is dominated by a single mode (Tierney and Kadane, 1986; Tanner, 1996). Bayesian posterior analysis requires the evaluation of integrals of the form, as shown in Eq. 29:

\[
I = \int g(\theta)e^{-nh(\theta)}d\theta
\]

where, \( g \) and \( -h \) are smooth functions of \( \theta \), with \( -h \) having a unique maximum at \( \hat{\theta} \). In Bayesian applications, \( -nh(\theta) \) may be the log-likelihood function or logarithm of the un-normalized posterior density \( p(\theta)L(y/\theta) \) and \( \hat{\theta} \) may be the maximum likelihood estimate. If \( g(\theta) \) has a unique sharp maximum at \( \hat{\theta} \), then most contribution to the integral \( I \) comes from the integral over a small neighborhood \( (\hat{\theta} - \delta, \hat{\theta} + \delta) \) of \( \hat{\theta} \) (Ghosh et al., 2006).

As \( n \to \infty \), we have, \( I = I_1 = \int_{\hat{\theta} - \delta}^{\hat{\theta} + \delta} g(\theta)e^{-nh(\theta)}d\theta \)

Laplace method involves Taylor series expansion of \( g \) and \( h \) about \( \hat{\theta} \), which gives,
Assuming that \( c = h''(\hat{\theta}) \) is positive and, using a change of variable, \( t = \sqrt{nc}(\theta - \hat{\theta}) \),

\[
I_1 \sim e^{-nh(\hat{\theta})} \frac{1}{\sqrt{nc}} \int_{-\delta / \sqrt{nc}}^{+\delta / \sqrt{nc}} \left[ 1 + \frac{t}{\sqrt{nc}} \frac{g'(\hat{\theta})}{g(\hat{\theta})} + \frac{t^2}{2nc} \frac{g''(\hat{\theta})}{g(\hat{\theta})} \right] \exp\left[ -\frac{t^2}{2} \right] dt
\]  

\[
= e^{-nh(\hat{\theta})} \sqrt{\frac{2\pi}{nc}} g(\hat{\theta}) [1 + O(n^{-1})]
\]  

The approximation for estimating the mean, \( E[g(\theta) / y] \) (Tierney and Kadane, 1986):

First apply the Laplace method to the numerator of Eq. 29 with \( g(\theta) \) positive, and, 

\[-nh^*(\theta) = -nh(\theta) + \log(g(\theta))\]  

(34)

where \( \theta^* \) is the mode of \(-nh^*(\theta)\) and,  

\[ \sigma^2 = \left[ \frac{\partial^2 - nh^*(\theta)}{\partial \theta^2} \right]_{\theta^*}^{-1/2} \]

Next, apply the Laplace method to the denominator of Eq. 29 with, \( g(\theta) = 1 \).

\[-nh(\hat{\theta}) = \log L(y / \theta) + \log(p(\theta))\]  

(35)

where \( \hat{\theta} \) is the mode of \(-nh(\theta)\) and,  

\[ \hat{\sigma}^2 = \left[ \frac{\partial^2 - nh(\theta)}{\partial \theta^2} \right]_{\hat{\theta}}^{-1/2} \]  

Taking the ratio, an approximate mean may be obtained as (Tierney and Kadane, 1986):
\[ E[g(\theta)] = \bar{\mu} = \frac{\sigma^*}{\sigma} \{\exp[-nh^*(\theta^*)]\}/\{\exp[-nh(\theta)]\} \]  

(36)

The simplest way to obtain such an approximation for posterior variance is to set:

\[ V[g(\theta)] = \bar{\sigma}^2 = E[g(\theta)^2] - E[g(\theta)]^2 \]  

(37)

Equation (4.16) may be used to approximate posterior means of \( g(\theta) \) and \( g(\theta)^2 \) and insert these values into a standard computational formula for variance (Eq. 37). Further, it is showed that the mean and variance have a relative error of (Tierney and Kadane, 1986):

\[ E[g(\theta)/y] = E(\hat{g})[1 + O(n^{-1})] \]  

(38)

\[ V[g(\theta)/y] = V(\hat{g})[1 + O(n^{-2})] \]  

(39)

Computational requirements of this approach are minimal; just need to evaluate the first and second derivative and maximize both the integrands. Still, the resulting approximations are quite accurate. An intuitive explanation for this is given (Tierney and Kadane, 1986); if the function is bounded away from zero near the posterior mode, then the two integrands will be similar in shape. Thus, by applying the same approximation technique to the numerator and the denominator one will be making similar errors, and in taking ratio some portion of these errors will be cancelled. Detailed mathematical derivation of the Laplace approximation for estimating parameters of the posterior distributions of known conjugate pairs, such as normal-normal, gamma-gamma, gamma-normal and gamma-poison may be found in Thodi et al. (2010).

5.5.3 **Comparison with conjugate pairs:** Both the M-H algorithm and Laplace approximation are coded in Matlab and used for developing the posteriors of the aforementioned degradation priors. The known conjugate prior-posterior values are used to validate the code. Inputs to the Matlab codes are the sampling size, the respective prior and likelihood parameters and the outputs are the estimated posterior parameters using the both methods. The sample prior, likelihood and conjugate posterior parameters considered are shown in Table 14a and the corresponding parameters estimated by the M-H algorithm and the Laplace approximation methods are presented in Table 14b. It has been observed that the M-H algorithm produced better results (Error < 12%) compared with Laplace approximations (Error < 28%). The error in Laplace estimation has been found to increase while estimating variances using higher order terms.
Table 14a: Parameters of prior, likelihood and conjugate pair posterior distributions.

<table>
<thead>
<tr>
<th>Prior distribution</th>
<th>Likelihood distribution</th>
<th>Posterior by conjugate pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Par1</td>
<td>Par2</td>
</tr>
<tr>
<td>Normal</td>
<td>5.00</td>
<td>2.00</td>
</tr>
<tr>
<td>Gamma</td>
<td>0.10</td>
<td>0.025</td>
</tr>
<tr>
<td>Gamma</td>
<td>0.10</td>
<td>0.025</td>
</tr>
<tr>
<td>Gamma</td>
<td>0.10</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Table 14b: Comparison of posteriors by M-H algorithm and Laplace approximations

<table>
<thead>
<tr>
<th>Posterior</th>
<th>M-H algorithm</th>
<th>Percentage error</th>
<th>Laplace appx.</th>
<th>Percentage error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Par1</td>
<td>Par2</td>
<td>Par1</td>
<td>Par2</td>
</tr>
<tr>
<td>Normal</td>
<td>8.2286</td>
<td>0.7498</td>
<td>-0.35</td>
<td>6.28</td>
</tr>
<tr>
<td>Gamma</td>
<td>2.3557</td>
<td>1.0362</td>
<td>-12.18</td>
<td>-1.09</td>
</tr>
<tr>
<td>Gamma</td>
<td>1.5616</td>
<td>0.7444</td>
<td>-4.11</td>
<td>0.75</td>
</tr>
<tr>
<td>Gamma</td>
<td>1.1986</td>
<td>1.0756</td>
<td>-8.96</td>
<td>-4.94</td>
</tr>
</tbody>
</table>

The prior-posterior analysis results obtained using the M-H algorithm for corrosion and cracking are summarized in Table 14, and are shown graphically in Figures 17 to 31. The M-H algorithm coded in Matlab has been used to simulate the posterior samples and to estimate their parameters. The posterior estimation based on M-H algorithm converges to results with around 10000 samples. First half of the simulated samples were ignored. These samples describe the transient state. The remaining samples which describe a steady state condition were used. The acceptance rate was above 65% which satisfies the statistical requirements. Being computationally intensive, the Laplace approximation was not very useful while using distributions with more than two parameters. The error in Laplace estimation has been found to increase as a result of computing the variance using second order terms. Since these posterior models are based on real life NDT data, they provide more reliable and accurate predictions for the future degradations of assets.
The use of a simulation method is necessitated because none of the prior models falls into the natural conjugate pair of the exponential family. Further, it has been observed that, for posterior estimation, the rejection sampling based M-H algorithm is the best method compared with the Laplace approximation method. Laplace approximation diverges as the parameter is either too small or too large due to numerical instability resulting from the use of higher order terms in the estimate. Therefore, for developing the posteriors of structural degradations in process components, the Laplace approximation would not be recommended. While using the M-H algorithm, the change in location parameter from priors to posteriors was found insignificant. Therefore, instead of using three-parameter models, the two-parameter models may be used to develop the posteriors and the location parameter may subsequently be added.

![Distribution Plot](image)

**Fig. 17**: Sample prior-posterior (Weibull) analysis result for UC (M-H algorithm)
Fig. 18: Sample prior-posterior analysis result for PC (M-H algorithm).

Fig. 19: Sample prior-posterior (Weibull) analysis result for EC (M-H algorithm)
Fig. 4.20: Sample prior-posterior (Weibull) analysis result for SCC (M-H algorithm)

Fig. 4.21: Sample prior-posterior analysis result for CFC (M-H algorithm)
5.6 Consequences analysis

To provide a consistent measure of risk, all consequence categories should be in the same units. Otherwise, the overall risk from many contributing sources cannot be computed. A standard choice of unit to represent all consequence categories is dollar, because risk can be interpreted as the expected loss due to a certain event or groups of events (Jones, 1995). Therefore, the failure consequences are expressed in terms of dollar in this project. The overall framework for economic consequence analysis is presented in Figure 22. The failure consequences include the costs of failure, inspection and maintenance.

5.6.1 Cost of failures: The operating and maintenance costs increase throughout the life of a process facility due to various degradation processes. Failure cost is the cost associated with the loss of a facility due to degradations. The failure cost may be divided into corrosion and cracking costs. Typically, consequence cost is equal to the sum of the failure costs, operating costs, and the cost of lost production, together with the material salvage value. It is assumed that a component failure is followed by an immediate repair to prevent any system failure scenario with much higher consequences. Degradation-related failures may lead to increased risk of loss of the entire unit through a chain of reactions. In such cases event tree analysis will be required to assess the system-level consequences. In this project, the component is assumed to be independent and isolated. The economic consequences of a component failure include loss of commodity due to breakdown, production loss due to shutdown, cost of spill cleanup, legal fees and penalties due to environmental damage and liability (Figure 22).

5.6.1.1 Loss due to breakdown: The leak or rupture of the component’s wall thickness by degradation is a main cause for breakdown. Hence, the breakdown costs are financial losses, which are associated with losing the commodity. This cost depends upon what product is being processed, the rate of leakage and its current market value when the failure occurs. The focus in this project is on a topside process piping unit in the North Sea, exporting crude oil. The market value of crude oil is assumed to be $70 per barrel in this
To estimate the rate of leakage, the source model, that is, the flow of liquid through a hole in a pipe, is used. The following formula may be used to estimate the cost of breakdown (Jackson, 2003):

\[
C_{f_{lp}} = E \times P \times D_{rp} \times Q_{pl} \times C_{dp}
\]

(40)

where, \(C_{f_{lp}}\) = the cost of the lost commodity in dollars, \(C_{dp}\) = cost of downtime calculated in dollars per barrel, \(Q_{pl}\) = quantity of commodity loss per unit of time (for e.g., barrels per hour), \(D_{rp}\) = duration of the commodity loss (hours), \(P\) = probability of loss of the commodity (depending on the equipment redundancy levels)=1 (assuming there is no redundancy and the components are in series), \(E\) = average number of critical failures in the lifetime. Estimated cost of piping degradation is presented in Table 15.

![Fig. 22: The Framework for Economic Consequence Analysis](image-url)
5.6.1.2 Loss of production due to shutdown: The major factor influencing the cost of failure is the facility’s unavailability for production. Inspection and maintenance can be planned, whereas failures may lead to an unplanned, immediate shutdown of the facility. The cost of such a shutdown is dependent on the number of days of shutdown, the rate of loss of production and value of products at the time of failure. Thus, shutdown cost is calculated by combining the unit cost of the product, loss of affected production and maintenance delay time as (Straub et al., 2006):

\[ C_{fd} = C_u \times Q \times T_m \]  

(41)

where, \( C_{fd} \) is the cost of shutdown (dollars), \( C_u \) is the unit cost of product (dollars/barrel), \( Q \) is the quantity of affected production (barrels/day) and \( T_m \) is the maintenance delay (days). The estimated cost of shutdown is presented in Table 15.

5.6.1.3 Cost of spill cleanup: The cost of an oil spill cleanup depends on a number of factors, such as, the type of oil, the amount spilled and rate of spillage, the characteristics of the affected area, weather and sea conditions, local and national laws, time of the year and the spill cleanup strategy (White and Molloy, 2003; Etkin, 2000). Predicting the unit cost of spill response is highly uncertain since the factors impacting the cost are complex. In the present article, crude oil spillage in offshore is considered. Based on the location, the average per-unit offshore oil spill cleanup cost is $6508 per tonne (Etkin, 2000). The cost of environmental cleanup comprises the unit cost of spill cleanup and the total quantity released due to failures caused by degradations. Further, the total quantity released depends on the rate of spillage and the duration of the release. The following formula may be used to estimate the cost of spill cleanup:

\[ C_{fc} = Q_m \times D_r \times C_{asc} \]  

(42)

where, \( C_{asc} \) is the unit cost of spill cleanup (dollars/tonne), \( Q_m \) loss of product per unit time (tonne/hour) due to corrosion or cracking, and \( D_r \) is the duration of spillage (hour). The cleanup cost thus estimated is presented in Table 15.

5.6.1.4 Loss due to environmental damage: The size of penalty as a result of damaging the environment is difficult to estimate, because costs increase with the scope of failure. The failure modes developed could escalate into more complex system failures leading to significant environmental damages. However, approximate assessments considering the quantity released and the unit penalty rate are possible. The environment damage due to oil spillage includes loss of marine as well as coastal habitat, soil pollution, damage to agriculture land and adverse health impacts (Etkin, 2000; Purnell, 1999). The per-unit cleanup cost of environmental damage is $ 5086 per tonne of oil (Etkin, 2000). This cost includes the cleanup cost of damage to the coastal ecosystem, consisting of near shore and shoreline response. The cost of environmental damage comprises the unit cost of nature damage and the total quantity released. The total quantity released depends on the rate of release and the duration of spillage. Thus, the total cost associated with damaging natural resources by failures may be estimated using the formula:

\[ C_{fnr} = Q_m \times D_r \times C_{dar} \]  

(43)

where, \( C_{dar} \) is the unit cost of nature damage (dollars/tonne), \( Q_m \) is the release of product per unit time (tonnes/hour) due to corrosion and cracking, and \( D_r \) is the duration of the release (hour). The nature damage cost due to degradation is presented in Table 15.

5.6.1.5 Cost of liability: The injuries and deaths caused by process component failure have the most severe implications possible. The loss of life or pain of an injury is impossible to quantify, yet, the cost
incurred due to workers compensation and corporate liabilities shall be taken into account (Jones, 1995). Apart from that, safety-related system failures have other immediate implications, such as legal fines and penalties for professional negligence. The estimates of liability costs that result from motor vehicle accidents are routinely published by several public and private organizations. The US Department of Transportation published a technical note (Judycki, 1994) on comprehensive motor vehicle accident costs which is adopted as a baseline in this article. The comprehensive liability cost includes medical costs, emergency services, vocational rehabilitation, lost earnings, administrative costs, legal consulting fees, pain and lost quality of life. For a typical piping failure, the liability is assumed to be a moderate injury, causing a lump sum payout of $40,000 in this project.

### 5.6.1.6 Total cost of failure

The total cost of failure \( C_f \) is the summation of loss of breakdown, loss due to shutdown, cost of spill cleanup and costs of environmental damage and liability, as:

\[
C_f = C_{fp} + C_{fsd} + C_{fse} + C_{fnr} + C_{fl}
\]

(44)

This total cost is based on two assumptions: the component is isolated, and the component failure leads to a system failure with subsequent unavailability. The estimated values for failure cost are presented Table 15. The rate of failure cost due to degradations, over the service life of \( n \) years, with varying inspection and maintenance intervals may be calculated using the following equation:

\[
FC(j) = C_f \frac{j}{n}
\]

(45)

where, \( j \) is the inspection and maintenance interval, which varies from 1 to \( n \) years.

#### Table 15: Degradation failure cost for piping components.

<table>
<thead>
<tr>
<th>Cost consequence</th>
<th>Cost divisions</th>
<th>Cost of corrosion (dollar)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loss due to breakdown</td>
<td>14,939</td>
<td></td>
</tr>
<tr>
<td>Loss due to shutdown</td>
<td>149,384</td>
<td></td>
</tr>
<tr>
<td>Spill cleanup</td>
<td>190,336</td>
<td></td>
</tr>
<tr>
<td>Damage to nature</td>
<td>148,748</td>
<td></td>
</tr>
<tr>
<td>Liability charges</td>
<td>40,000</td>
<td></td>
</tr>
<tr>
<td>Total cost ( C_f )</td>
<td>543,77</td>
<td></td>
</tr>
</tbody>
</table>

### 5.6.2 Consequences of inspection

The NDT techniques are used for the detection and quantification of unwanted discontinuities and separations in materials due to degradations. This quantitative information is achieved by detecting, locating and sizing of any detected flaws. Several types of defects exist in components, such as corrosion, cracking, inclusions, dents and holes. Defect quantification requires considerable skill and experience, and the use of more than one NDT technique. Based on literature (Roberge, 2007; Gros, 1997), the best suitable inspection methods for corrosion and cracking are identified, and their corresponding dollar costs are estimated. The unit costs of the NDT techniques obtained from an inspection contracting company have been used in the analysis.
5.6.2.1 **Cost of degradation inspection:** The NDT technique is used to detect and quantify the extent of wall loss, pit depth and surface cracks as well as coating breakage. The inspection costs depend on how much area to inspect from a risk perspective. The inspection cost includes the cost for gaining access to the degraded component, the cost for surface preparation, personnel cost for inspection, the cost associated with technical assistance, the cost of consumables and chemicals, and the logistics cost. In this article, it is assumed that the proposed inspection method is able to detect the presence of corrosion discontinuities, and surface or subsurface cracks. For piping (pipeline segments, bends and tees), the suggested inspection methods are UT thickness measurement and radiographic inspection (RI) for corrosion, and magnetic particle inspection (MPI) and UT defect sizing for cracking. The cost of each inspection activity is estimated using the per-unit personnel cost, and the total duration of inspection. Cost associated with piping inspection \( C_I \) is:

\[
C_I = C_{iga} + C_{isp} + C_{utt} + C_{ir} + C_{ita} + C_{il}
\]

where, \( C_{iga} \) = cost of gaining access, \( C_{isp} \) = cost of surface preparation, \( C_{utt} \) = cost of UT defect sizing, \( C_{ir} \) = cost of radiographic inspection, \( C_{ita} \) = cost of technical assistance and \( C_{il} \) = cost of logistics (equipment storage, rent and transportation). The cost of UT thickness measurements, \( C_{utt} = C_{utt} \times t \), whereas \( C_{utt} \) = personnel cost for UT thickness measurements per hour, and \( t \) = total duration of inspection in hours. The estimated costs for corrosion and cracking are presented in Table 16. On an annual basis, the rate of inspection costs tends to decrease with the increase in inspection and maintenance intervals. This decreasing trend may be modeled using the following equation:

\[
IC(j) = C_I \frac{n}{j}
\]

where, \( j \) is the inspection interval, \( IC(j) \) is the inspection cost in the \( j^{th} \) interval, and \( n \) is the component service life in years.

5.6.3 **Economic consequences of maintenance:** This is the cost associated with restoring degraded/failed components. To ensure safe operation, maintenance needs to be performed at very small intervals. However, it is impractical to have frequent maintenance due to large costs, the possibility of maintenance-induced errors, and the associated plant unavailability. To optimize maintenance, the following necessary conditions must be satisfied: the cost of maintenance should be greater after failure than before, and the hazard rate of the component should be increasing, i.e., the component should be in the wear-out region. This project focuses on predictive maintenance of process components. Predictive maintenance estimates through diagnostic tools, such as NDT techniques and probabilistic models, when a component or part is about to fail and should be repaired or replaced; thus reducing costly corrective maintenance. It covers the cost of necessary repair, replacement, and material costs associated with inspection and maintenance.

5.6.3.1 **Cost of degradation maintenance:** Maintenance may be either a minor patch repair task or the complete replacement of a degraded component. For all types of corrosion, minor patch repair work of the affected area is considered, and for any types of cracking, intermediate component replacement with necessary repair is considered. Maintenance task includes access to the degraded part, surface preparation, cutting and removal of parts, assembling, welding, testing and restoring the protective coating. Thus, in addition to the cost of repair and replacement, the personnel and logistics cost related to transportation, storage and rent of facilities also should be included. The cost of each maintenance activity is estimated using the unit cost of maintenance personnel and the total duration of maintenance. Details of the
estimation have been presented in Thodi et al. (2011). The total cost associated with piping maintenance for degradation may be estimated as:

\[ C_M = C_{mga} + C_{msp} + C_{mgd} + C_{mmr} \]  \hspace{1cm} (48)

where, \( C_{mga} \) = cost of gaining access to the degraded component, \( C_{msp} \) = cost of surface preparation, \( C_{mgd} \) = cost of gouging defects, and \( C_{mmr} \) = cost of minimal repair or replacement. Where, the repair (cutting, welding and fitting) cost, \( C_{mcw} = C_{icr} \times t \), whereas the \( C_{icr} \) is cost of labor for repair in dollars per hour, \( t \) is the total repair time in hours. The rate of maintenance costs decreases with the increase in maintenance intervals over the service life. This decreasing trend may be modeled using the following equation:

\[ MC(j) = C_M \frac{n}{j} \]  \hspace{1cm} (49)

where, \( j \) is the inspection interval, \( MC(j) \) is the maintenance cost for the \( j^{th} \) interval, and \( n \) is the service life in years. The cost estimates associated with piping degradation (corrosion and cracking) are presented in Table 16.

5.6.4 Annual equivalent cost of degradations: The annual equivalent cost (AEC) of operating and maintaining the component is the summation of the rate costs of failure, inspection and maintenance, and is estimated as:

\[ AEC(j) = FC(j) + IC(j) + MC(j) \]  \hspace{1cm} (50)

Due to the increasing trend of rate of failure cost and the decreasing trends of rate of inspection and maintenance costs, the AEC v/s maintenance interval will be a convex function.

5.6.5 Probabilistic cost analysis: Uncertainty and variability in consequence analysis are modeled with probabilistic analysis using Monte Carlo simulations. For simulation, the total cost of component failure, inspection and maintenance is considered to be a Gaussian distribution with the estimated mean. The coefficients of variation of costs are assumed to be 2.5%. The estimated mean and standard deviation values of the piping degradation costs are reported in Table 15.

Table 15. Probabilistic Piping Degradation Costs used in the Economic Analysis

<table>
<thead>
<tr>
<th>Structural degradation &amp; Cracking (SCC, CFC, HIC)</th>
<th>Cost divisions</th>
<th>Corrosion cost ($)</th>
<th>Cracking cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. dev</td>
<td>Mean</td>
</tr>
<tr>
<td>Total cost of failure</td>
<td>543 407</td>
<td>13585</td>
<td>438 235</td>
</tr>
<tr>
<td>Total cost of maintenance</td>
<td>10 000</td>
<td>250</td>
<td>15 000</td>
</tr>
<tr>
<td>Total cost of inspection</td>
<td>3840</td>
<td>96</td>
<td>4 400</td>
</tr>
<tr>
<td>Salvage value</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Annual rate of interest</td>
<td>8 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Service period</td>
<td></td>
<td></td>
<td>30 years</td>
</tr>
</tbody>
</table>
5.7 Optimization of inspection and maintenance

The AEC has been combined with the cumulative density function (CDF) of the posterior probability to estimate the operational life risk as shown in Eq. 51. Thus, finding the optimal inspection and maintenance interval is reduced to finding the value of inspection and maintenance intervals that minimizes the operational risk. At the optimal risk point, the risk will be reduced to as low as reasonably practicable (ALARP) level, which at the same time ensures the safety of the facility’s operation.

\[ R(j) = F[p(\theta / y), j] \times AEC(j) \]  
\[ (51) \]

where, \( R(j) \) is the risk of failure due to degradation (in dollars) in the \( j^{th} \) interval, \( F[p(\theta / y), j] \) is the CDF of posterior probability of failure and \( AEC \) is the annual equivalent cost, corresponding to the inspection and maintenance interval, \( j \). The operational risk curve is observed to be a convex function of the component’s service life. A search is conducted to identify the minimum risk point, and the interval of this minimum risk is considered as the optimal inspection and maintenance interval. The optimum inspection and maintenance interval thus obtained satisfies the two necessary criteria of optimal maintenance: one, the risk is reduced to ALARP level; and two, the maintenance interval is maximized, thus avoiding unwanted maintenance and its associated costs. The inspection and maintenance risk in dollars is compared with the company’s operating budget, as risk acceptance criteria. The results of estimated risk due to UC, PC, EC, SCC, CFC, and HIC of process components are discussed in Section 5.9.

The sample results of operational life risks due to corrosion and cracking are presented in Figures 4.15 to 4.20. These Figures show overall risk in dollars due to various structural degradations, such as UC, PC, EC, SCC, CFC and HIC, plotted against the inspection and maintenance interval. On the risk curve thus developed, the point where the risk is minimal is defined as the optimum maintenance interval for the component with respect to that particular degradation process. The degradation processes are assumed to be independent of each other and isolated. Also, it is assumed that the minimal repair for corrosion leaves the system in a state similar to its state just before its failure, whereas the replacement for cracking brings the system back to an as good as new condition. With respect to the considered piping degradations, the computed optimal inspection and maintenance intervals are reported in Table 17. The optimum maintenance interval is the time to the next inspection and maintenance starting from now onwards. Around 10 000 iterations are used to produce operational risk curves, shown in Figures 24 to 29.

5.8 Optimization of replacement

The AEC has been combined with the cumulative density function (CDF) of the posterior probability to estimate the operational life risk curve as shown in Eq. 40. Thus, finding the optimal replacement period reduces to finding a value of \( n \) that minimizes the operational risk. At the optimal risk point, the risk will be reduced to as low as reasonably practicable (ALARP) level and at the same time, ensures the safety of operation.

\[ R = F[p(\theta / y)] \times AEC \]  
\[ (52) \]

where, \( R \) is the risk of failure (in dollar) from a degradation, \( F[p(\theta / y)] \) is the CDF of posterior probability of failure and \( AEC \) is the annual equivalent cost of consequences.
Replacement strategies are designed to remedy the effects of physical deterioration, strength loss and obsolescence of process components. Physical deterioration leads to reduction in the efficiency of operation, wall thickness and material strength. Obsolescence occurs as a result of continuous developments of new component. The annual equivalent cost is calculated by combining the failure, inspection and maintenance costs. The optimal replacement interval is the interval corresponding to minimum risk. By performing replacement at this interval, the risk of operation will be reduced to the ALARP level. In this project, a case study of a pipeline segment was presented. The optimum replacement intervals for a pipeline segment were found to be 10 years for EC and 8 years for CFC. The smaller of these two values has been selected as the optimum replacement interval for the ageing pipeline segment. The replacement strategy entails the economic replacement of components rather than performing maintenance. This model takes into account the effects of taxes, the uncertainty and variability in the degradation process and the consequence parameters using the Bayesian Monte Carlo simulations.
Fig. 24: Operational life risk curve due to uniform corrosion.

Fig. 25: Operational life risk curve due to pitting corrosion.
Fig. 26: Operational life risk curve due to erosion corrosion.

Fig. 27: Operational life risk curve due to stress corrosion cracking.
Fig. 28: Operational life risk curve due to corrosion fatigue cracking.

Fig. 29: Operational life risk curve due to hydrogen induced cracking.
5.9 Discussion

The sample results of operational life risks due to corrosion and cracking are presented in Figures 24 to 29. These Figures show overall risk in dollars due to various structural degradations, such as UC, PC, EC, SCC, CFC and HIC, plotted against the inspection and maintenance interval. On the risk curve thus developed, the point where the risk is minimal is defined as the optimum maintenance interval for the component with respect to that particular degradation process. It is assumed that the minimal repair for corrosion leaves the system in a state similar to its state just before its failure, whereas the replacement for cracking brings the system back to an as good as new condition. With respect to the considered piping degradations, the computed optimal inspection and maintenance and replacement intervals are reported in Table 16. The optimum maintenance interval is the time to the next inspection and maintenance, and the optimum replacement interval is the time to replace the component starting from now onwards. Around 10 000 iterations are used to produce operational risk curves, shown above.

Table 16: Optimal Interval for the Maintenance and Replacement of Components

<table>
<thead>
<tr>
<th>Process Component</th>
<th>Deterioration Process</th>
<th>Degradation Model</th>
<th>Replacement Interval (years)</th>
<th>Maintenance Interval (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Piping (straight pipes, bends, tees)</td>
<td>UC</td>
<td>3P Weibull</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>PC</td>
<td>Type 1 Ex Val.</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>EC</td>
<td>3P Weibull</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>SCC</td>
<td>Weibull</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>CFC</td>
<td>Weibull</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>HIC</td>
<td>Weibull</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

6. Development of a methodology for risk-based inspection and maintenance planning

Risk-based inspection and maintenance (RBIM) is a strategy which aims at maintaining the integrity of a plant or an asset that is subjected to deterioration. The developed methodology comprises of the following main parts:

6.1 Inspection sampling

Often, it is impractical to inspect the large number of components that constitute a complete system. Therefore, only a sample of the components is inspected. In this case, it is required that the results obtained from the sample best represent the population.

Some standard codes such as API 570, 1998 (piping inspection code), API 510, 1998 (pressure vessels inspection code) and API 581, 2000 (Risk Based Inspection) provide excellent recommendations for conducting inspection and determining frequency of inspections, however, these codes do not provide specific guidelines to determine the inspection sample size to represent the condition of the complete system. For example, in API 570, inspection sample size is left for the inspection practitioner’s judgment (Hobbs and Ku, 2002). API 581 (risk-based inspection) considers risk as a basis for inspection planning.
Although API 581 provides guidance for determining frequency of inspections, it does not provide specific guidance for determining minimum number of locations to be inspected to represent the condition of the complete system.

**6.1.1 General corrosion:** A Bayesian-based approach is developed for calculating the minimum size of a sample to assess, with a specified precision, the integrity of process components suffering from general corrosion. An analytical formula to estimate the sample size is introduced. The sample size obtained using the developed approach is smaller than a sample size obtained using the classical approach with same confidence level. This reduces sampling inspection cost without affecting the precision of the estimate.

In engineering applications, decisions are often made based on whatever limited available information. The available information before performing new inspection is referred to as prior information. This information could be based on historical data, theoretical models or expert subjective judgement. One can use newly obtained inspection data to update prior information using Bayesian updating theory. The updated information is referred to as posterior information. For example, prior information of metal loss of equipment can be updated once a new set of inspection data is obtained.

Let $x$ denotes observed independent sample data. Assume the distribution of the random variable, $x$, has a parameter $\theta$. Assume further, that the parameter $\theta$ is a random variable having a known prior probability distribution $f'(\theta)$. The prior distribution of the parameter $\theta$ can be updated using Bayes’ theorem as follows:

$$f''(\theta) = \frac{P(x|\theta) f(\theta)}{\int_{-\infty}^{\infty} P(x|\theta) f(\theta) \, d\theta}$$

(53)

where $f''(\theta)$ is the posterior (updated) probability distribution of $\theta$ and $P(x|\theta)$ is the conditional probability of observing the inspection outcome $x$ for a given $\theta$ and commonly referred to as the likelihood function of $\theta$.

The posterior estimate, $\theta''$, for the mean of the parameter $\theta$ is given by:

$$\theta'' = \int_{-\infty}^{\infty} \theta \ f''(\theta) \, d\theta$$

(54)

Assume the parameter $\theta$ to denote the sample mean of the corrosion depth and assume that it is normally distributed. Further assume that the prior distribution of the sample mean is normal. In this case, the posterior distribution of the sample mean is also normal. The posterior estimates for the mean and the standard deviation of the sample mean are obtained from Bayes’ theorem as follows (Ang and Tang, 2007):

$$\mu'' = \frac{\mu'_x (c'_x)^2 + \mu (\frac{\sigma^2}{n})}{(c'_x)^2 + (\frac{\sigma^2}{n})}$$

(55)
\[
\sigma'_{\mu s} = \sqrt{\frac{(\sigma'_{\mu s})^2 + (\frac{\sigma_s^2}{n})}{(\sigma'_m)^2 + (\frac{\sigma_s^2}{n})}}
\]  
(56)

where \(\sigma'_{\mu s}\) is the prior estimate of the standard deviation of the sample mean and \(\sigma_s\) is the standard deviation of the sample of size \(n\). \(\mu'\) is the prior mean.

If all possible random samples, each of size \(n\), are drawn from a prior distribution with standard deviation of \(\sigma'\) and the mean of each sample is estimated, the prior standard deviation of the sample mean, \(\sigma'_{\mu s}\), is given by:

\[
\sigma'_{\mu s} = \frac{\sigma'}{\sqrt{n}}
\]  
(57)

Substitute for \(\sigma'_{\mu s}\) in Eq. 55 and Eq. 56, one gets:

\[
\mu''_{\mu s} = \frac{\mu_{\mu s}' + \frac{\sigma'_{\mu s}^2}{\sigma^2 + \frac{\sigma_s^2}{n}}}{\sigma^2 + \frac{\sigma_s^2}{n}}
\]  
(58)

\[
\sigma''_{\mu s} = \sqrt{\frac{\left(\frac{\sigma''_{\mu s}}{n}\right)\left(\frac{\sigma_s^2}{n}\right)}{\left(\frac{\sigma^2}{n}\right) + \left(\frac{\sigma_s^2}{n}\right)}}
\]  
(59)

\[
\sigma''_{\mu s} = \frac{\sigma''_{\mu s}}{\sqrt{n}}
\]  
(60)

where \(\sigma''_{\mu s} = \sqrt{\frac{\sigma^2 - \frac{\sigma_s^2}{n}}{\sigma^2 + \frac{\sigma_s^2}{n}}}\)

Eqs. 58 and 59 give expressions for the posterior estimates of the sample mean and standard deviation in terms of the new and prior information.

When prior information and new sample inspection data are available, the population’s properties can be estimated on the basis of the posterior estimates for the parameters. For uninformative prior information (i.e., \(\sigma'\) approaches infinity), from Eqs. 55 and 56, the limit of \(\mu''_{\mu s}\) approaches the sample mean, \(\mu_s\) and the limit of \(\sigma''_{\mu s}\) approaches \(\sigma_s/\sqrt{n}\) as in the classical approach.

For a finite population, \(\sigma''_{\mu s}\) is obtained using the finite population correction factor and Eq. 61 as follows:
The margin of error of the posterior sample mean can be defined as follows:

\[ \text{MOE}_{\text{mean}} = \Phi^{-1}(1 - \alpha/2), \sigma_{\mu s} \]  

Using Eqs 63 and 64, a formula to estimate the required sample size \( n \) to evaluate the posterior mean of a population of size \( N \) with pre-defined acceptable margin of error (MOE_{accept}) can be obtained as

\[ n = \left[ \frac{\Phi^{-1}(1 - \alpha/2)}{\sigma_{\text{MOE accept}}} \right]^2 \frac{1}{\Phi^{-1}(1 - \alpha/2)(\sigma_{\mu s}/N)^2} \]  

For more details and application of the developed Bayesian approach-based method for calculating sample size to assess general corrosion, see Khalifa, et al. 2011a.

6.1.2 Localized corrosion: There is a lack of studies aiming to address calculation of the localized corrosion sample size and still there is no clear consensus on this problem. However, it is widely believed that the larger the sample size, the smaller the error of the sample estimate (Kowaka et al., 1984 and Alfonso et al., 2008). This problem was studied previously by Shibata (1991), Schneider et al. (2001), Wang (2006) and Alfonso et al. (2008). Shibata (1991) showed from historical data that localized corrosion may be modeled with Gumbel extreme value distribution with location parameter (\( \lambda \)) and scale parameter (\( s \)) changing with time while the ratio (\( s/\lambda \)) remains approximately constant irrespective of the time for a certain material in same environment. Shibata (1991) plotted sample size (\( n \)) versus return period (\( T \)) for different ratios (\( s/\lambda \)) estimated using the minimum variance linear unbiased estimator method. From this plot, the optimum sample size is obtained for a given return period. Schneider et al. (2001) estimated the required inspection area by investigating the dependence between the data points at different distances from each other. Wang (2006) estimated the required number of tubes in heat exchangers for the assessment of the minimum remaining thickness of tubes in industrial heat exchangers subjected to corrosion. Alfonso et al. (2008) aimed to estimate the optimum sample size and unit inspection area in a long pipeline based on the required accuracy of the estimate of maximum pit depth in un-pigable, buried pipelines using the extreme value method. Alfonso et al. (2008) performed extensive Monte Carlo simulations to estimate the mean square error of the estimate of the maximum localized corrosion as an indicator to the estimate accuracy for different sample sizes.

The extreme value method is widely used to predict the maximum localized corrosion over the entire population using sample data. A major limitation of the application of the extreme value method to the assessment of the corrosion is that the sample size affects the accuracy of the extreme value prediction (Jarrah et al., 2011). An approach is developed to estimate the required sample size to assess, with a specified precision, the localized corrosion of process components. The developed approach uses the extreme value and bootstrap methods. The results of estimated sample size ensure that the predicted maximum localized corrosion with the extreme value method is within an acceptable margin of error at a
specified confidence level. The developed approach compromised the following main parts as shown on the following flowchart (Figure 30):
Fig. 30: Flowchart of the developed approach to estimate the required sample size to assess the localized corrosion.

**Part 1**
- i. Layering separation (separate the system into corrosion circuits/groups)

**Part 2**
- ii. Sample with size n from a group (population)
- iii. Fit Gumbel distribution to inspection data

**Part 3**
- iv. Generate a bootstrap population of size N following the fitted Gumbel distribution
- v. Let the bootstrap sample size = \( n_b \leq N \)
- vi. Draw a bootstrap sample without replacement from the bootstrap population
- vii. Fit Gumbel distribution to the drawn bootstrap sample
- viii. Predict the maximum corrosion with extreme value method
- ix. Calculate bootstrap standard error of the maximum (\( SE_{max} \))
- x. Estimate coefficient of error of the maximum (\( COE_{max} \))

**Part 4**
- xi. Average \( COE_{max} \) for each sample size \( n_b \leq N \), plot average \( COE_{max} \) versus \( n_b \) and obtain the sample size \( n_b \) corresponding to the acceptable \( COE_{accept} \).

- Increase n to \( n_b \)
- Is obtained \( n_b \geq n \)?
  - Yes
  - No

- xii. The number of all inspected components/areas is the required sample size
Part 1 - Layering separation: Through layering separation the equipment of an installation subjected to corrosion is classified into groups or areas. The groups obtained by this classification process are usually referred to as corrosion circuits or loops. A corrosion circuit (loop) is a group of similar assets in the plant which have identical material and are exposed to identical corrosion conditions. Each group is considered as a population from which sampling is required. The objective of layering separation is to reduce the source of variability in the inspection data within each group. This would help to reduce the required sample size when sampling randomly within a group because sample size is strongly dependent on the standard deviation, $\sigma$, of the population.

Part 2 - Physical sampling within each group: A randomly selected number of components/areas within a group is inspected. Only the maximum localized corrosion of each component/area is recorded and fitted to a Gumbel extreme value distribution.

Part 3 - Bootstrap sampling and extreme value analysis:

Use of bootstrap sampling methods to estimate standard error and confidence interval: The standard error is a measure of the accuracy of an estimator obtained based on sample data. There is no accurate formula for estimating the standard error of a statistic other than the mean.

In case of localized corrosion, as there is no close analytical expression to estimate the confidence interval and therefore the margin of error of the maximum localized corrosion, bootstrap sampling is used in the developed approach for this purpose.

Use of the extreme value statistical method to predict the maximum localized corrosion: The extreme value distribution is classified into three types (Type I, Type II and Type III) for two cases (maximum values and minimum values). Type I (in case of maximum values) is known as Gumbel distribution. It is a common practice to use Gumbel distribution to represent the probability distribution of maximum localized corrosion; see Kowako et al. (1984). The cumulative probability of Gumbel distribution is given by:

$$F(x) = \exp\left(-\exp\left(-\frac{x-\lambda}{\theta}\right)\right)$$  \hspace{1cm} (66)

where $F$ is the cumulative density function of the random variable $x_{\text{max}}$ (maximum value), $\lambda$ and $\theta$ are location and scale parameters, respectively. Several methods can be used to estimate $\lambda$ and $\theta$ such as the maximum likelihood method or fitting a straight line in the Gumbel probability paper.

The mean, $\mu$ and standard deviation, $\sigma$ are estimated in terms of the scale and location parameters as follows:

$$\mu = \lambda + \gamma \theta$$ \hspace{1cm} (67)

where $\gamma = 0.57722$ is Euler constant

$$\sigma = \frac{\pi}{\sqrt{6}} \theta$$ \hspace{1cm} (68)

The localized corrosion may be modeled by extreme value distribution [Khan and Howard (2007)].
extrapolated for the whole population to predict the maximum corrosion size in uninspected areas (Kowaka et al., 1984; The Health and Safety Executive, 2002; and ASTM G46-94, 2005).

To demonstrate how the maximum corrosion is predicted over the whole population with the extreme value method, let us consider a system having a total of N components/areas (population of size N) and a sample of size n components/areas is inspected. The sample data points (measured maximum corrosion for each component/area) are arranged in order of increasing rank. The cumulative probability, F, can be calculated as i/(n+1), when using the average rank method, where i is the order of rank and n is the sample size (number of recorded maxima). The highest value of the sample cumulative probability can be estimated as F=n/(n+1). This maximum value corresponds to the maximum corrosion over the sample. Similarly, the highest value of the cumulative probability for the whole population of size N can be estimated as F=N/(N+1). The Gumbel extreme value cumulative probability function is a straight line on Gumbel probability plot paper. Thus, the maximum localized corrosion over the entire population can be predicted by extrapolating the Gumbel probability plot linearly from point A to point B (Figure 31).

**Fig. 31:** Extrapolation of the Gumbel probability plot to predict the maximum localized corrosion

The return period T of localized corrosion can be explained as number of unit inspection areas at which the maximum corrosion is observed. The scale of return period $T = 1/(1-F)$ is shown on the right-hand side of the vertical axis of Figure 31.

The extrapolation shown in Figure 31 is valid provided that the statistical characteristics of the sample completely represent the statistical characteristics of the whole population. Thus determining the sample size to represent the whole population is important to ensure the precision of the extreme value method in the prediction of the maximum corrosion.

In the developed approach, a large number of bootstrap samples of different sizes $n_b \leq N$ are drawn without replacement from generated bootstrap populations of the same size of the original population, N. The maximum localized corrosion is predicted with the extreme value method for each bootstrap sample. The bootstrap standard error of the maximum corrosion, $SE_{\text{max}}$, predicted with the extreme value method is estimated.
Part 4 - Calculation of the required sample size to predict the maximum localized corrosion within each group: It is required to estimate the appropriate sample size within each group which provides an accurate description of the state of the whole group.

In case of localized corrosion, evaluation of the mean is not sufficient because the failure is expected when the maximum corrosion at any location in the population exceeds the critical limit. Thus, the sampling objective is to predict the maximum corrosion, not the mean, over the whole population (inspected and uninspected components/areas). In order to achieve that, the following steps are undertaken:

i) Standard error of the population maximum (SE$_{\text{max}}$) is estimated as the standard deviation of the population maximum which is predicted using the extreme value method (see Figure 2) for large number of bootstrap samples of sizes $n_b \leq N$.

ii) The ratio of standard error of the population maximum (SE$_{\text{max}}$) to standard deviation of sample data ($\sigma_s$) is evaluated for each bootstrap sample. In this work, we will refer to this ratio as the coefficient of error of the population maximum (COE$_{\text{max}}$) and define it as follows:

$$\text{COE}_{\text{max}} = \frac{\text{SE}_{\text{max}}}{\sigma_s} \quad (69)$$

iii) The margin of error of population maximum, MOE$_{\text{max}}$, is expressed as half of the confidence interval of the maximum and is given by:

$$\text{MOE}_{\text{max}} = \Phi^{-1} \left( 1 - \frac{\alpha}{2} \right) \cdot \text{SE}_{\text{max}} \quad (70)$$

iv) From Eq. 15 and Eq. 16, the coefficient of error of the maximum, COE$_{\text{max}}$, is given by:

$$\text{COE}_{\text{max}} = \frac{\text{MOE}_{\text{max}}}{[\Phi^{-1} \left( 1 - \frac{\alpha}{2} \right) \cdot \sigma_s]} \quad (71)$$

v) The acceptable coefficient of error COE$_{\text{accept}}$ is estimated from Eq. 71 corresponding to a pre-defined acceptable margin of error, MOE$_{\text{accept}}$, as follows:

$$\text{COE}_{\text{accept}} = \frac{\text{MOE}_{\text{accept}}}{[\Phi^{-1} \left( 1 - \frac{\alpha}{2} \right) \cdot \sigma_s]} \quad (72)$$

vi) The bootstrap sample size, $n_b$, is plotted versus COE$_{\text{max}}$ estimated from Eq. 69. Then from this plot, the required sample size is obtained corresponding to the acceptable COE$_{\text{accept}}$.

vii) An equation is fitted to the results obtained with the developed approach as follows:

A. The sample mean and standard deviation are set at two levels (low and high) of 0.1 and 100 units. The scale and location parameters of Gumbel distribution are estimated for each sample.

B. Bootstrap populations were generated following the sample Gumbel distribution with size, $N$, at levels of $10^2$, $10^3$, $10^4$, $10^5$, $10^6$ and $10^7$.

C. The COE$_{\text{max}}$ is estimated for all possible combinations with different levels of sample mean ($\mu_s$), standard deviation ($\sigma_s$), population size (N), and bootstrap sample size $n_b \leq N$.

D. The estimated COE$_{\text{max}}$ is plotted versus bootstrap sample size $n_b$ for different levels of population size N. For example, Figure 32 shows COE$_{\text{max}}$ versus bootstrap sample size, $n_b$, for population size, N = 100 with two levels of $\mu_s$ and $\sigma_s$ are 0.1 and 100 units. For
example, Figures 32A and 32B show COE$_{\text{max}}$ versus bootstrap sample size, $n_b$, for population size, $N = 10^2$ and $N = 10^7$ respectively with two levels of $\mu_s$ and $\sigma_s$ are 0.1 and 100 units.

**Fig. 32A**: Bootstrap sample size, $n_b$, versus COE$_{\text{max}}$ for $N = 10^2$.

**Fig. 32B**: Bootstrap sample size, $n_b$, versus COE$_{\text{max}}$ for $N = 10^7$.

E. The results of COE$_{\text{max}}$ are fitted to the following equation:

$$
\text{COE}_{\text{max}} = f(N) \cdot \sqrt{\left(\frac{1}{n} - \frac{1}{N}\right)} , \quad N \leq 10^7 \tag{73}
$$

$$
f(N) = \begin{cases} 
1.3N^{0.2} & , N \leq 200 \\
2.1N^{0.13} & , 200 < N \leq 10^4 \\
3.2N^{0.083} & , 10^4 < N \leq 10^7 
\end{cases}
$$
This leads to:

\[ n = \frac{1}{(\frac{\text{COE}_{\text{max}}}{f(N)})^2 + \frac{1}{N}} \]  

(74)

From Eq. 72 and Eq. 74, the sample size required to predict the population maximum with pre-defined MOE_{\text{accept}} can be calculated using the following equation:

\[ n = \frac{1}{\left(\frac{\text{MOE}_{\text{accept}}}{f(N) \cdot \Phi^{-1}\left(1 - \frac{\alpha}{2}\right) \sigma_{x}}\right)^2 + \frac{1}{N}} \quad , N \leq 10^7 \]  

(75)

The estimated sample size using the proposed equation (Eq. 75) ensures that the predicted maximum localized corrosion using the extreme value method is within pre-defined \( \pm \text{MOE}_{\text{accept}} \) at a confidence level \( (1-\alpha) \).

F. The above steps are repeated for the mean instead of the maximum value. In this case, an equation similar to Eq. 75, with \( f(N) = 1 \), is obtained. It should be noticed that Eq. 75 yields to Eq. 65 when \( f(N) = 1 \) and no prior information is available.

For more details and application of the proposed approach to estimate the required sample size to assess the localized corrosion, see Khalifa et al., 2011b.

The formula given by Eq. 75 is limited by \( N \leq 10^7 \). In addition it was assumed there is no available prior information. Another two closed form formulas are obtained for \( N \leq \infty \) to address the two situations when prior information is available or is not available. Bayesian updating approach is used to update prior information obtained from a previous inspection and engineering judgement once newly obtained information is available from the current inspection. Figure 33 shows the flowchart of the developed approach to obtain these two formulas.
The obtained two formulas are as follows:

Case 1: Without having prior information
Case 2: With prior information

\[
MOE_{\text{accept}} = \phi^{-1}(1 - \frac{\alpha}{2}) \cdot MVCF \cdot \frac{\sigma^2 \left( \frac{1}{n} \right) \left( \frac{\sigma^2 \left( \frac{1}{n} \right) + 2\sigma^2 \left( \frac{1}{n} \right)}{\sigma^2 \left( \frac{1}{n} \right) + 2\sigma^2 \left( \frac{1}{n} \right)} \right)}{\sigma^2 \left( \frac{1}{n} \right) + 2\sigma^2 \left( \frac{1}{n} \right)} \frac{(n)}{N}
\]

where

- \( n \) is sample size in the current inspection
- \( n_o \) is sample size in the previous inspection
- \( \sigma \) is standard deviation of maximum corrosion size in each inspected component/area in the current inspection
- \( \sigma_o \) is standard deviation of maximum corrosion size in each inspected component/area in the previous inspection
- \( MOE_{\text{accept}} \) is acceptable margin of error
- \( MVCF \) is defined as maximum value correction factor and is given by:

\[
MVCF = \sqrt{1 + \frac{\sigma^2}{\pi^2 n \sqrt{\ln \left( \frac{N}{N+1} \right)}}}
\]

For more details of deriving these two formulas, see Khalifa et al., 2011c.

### 6.2 Inspection and maintenance planning

Inspection is carried out at pre-defined intervals and at prescribed locations. This monitoring strategy is needed to ensure that all assets perform their intended functions and that plant integrity is not threatened. An inspection and maintenance plan involves selection of the inspection interval and required maintenance strategy such as repair or replacement. The condition of the inspected asset dictates the maintenance action that needs to be taken. The asset may be repaired, replaced, or left as is depending on its condition. The action taken is based on the maintenance cost and the acceptable risk of failure of the asset to perform its intended function until the next inspection. A balance between the cost of inspection and maintenance and risk of failure is to be achieved to optimize the RBIM plan.

A quantitative risk-based inspection and maintenance (RBIM) approach for inspection and maintenance planning is developed. The developed approach aimed to maintain the integrity of assets subjected to deterioration in a cost effective manner.
The developed approach comprised of main steps: classification of asset's components/areas according to criticality of deterioration, asset failure modeling, risk assessment, cost estimation (inspection and maintenance) and optimal selection of inspection interval and maintenance strategy. To solve the optimization problem, an objective function is formulated as a function of the present value of inspection cost, repair/replacement cost, risk of failure and the remaining value of the asset after a specified period of time. The selection of the optimum inspection interval and maintenance activity is based on minimizing the objective function subject to a constraint that the risk of failure over the lifetime of the asset does not exceed an acceptable level. The developed approach allows minimizing the cost of inspection and maintenance over the lifetime of a deteriorated asset/system without compromising the safety.

The proposed methodology framework is shown in Figure 34.
Fig. 34: The developed methodology framework for RBIM.
6.1.1 Inspection interval

In the developed methodology, the inspection interval is selected as a fraction, \( R \), of the remaining life. \( R \) is a controllable (decision) variable in the optimization problem and is determined on the basis of the optimization of the RBIM plan. Figure 35 shows the inspection intervals between the initial time, \( t_0 \), and the critical time to failure, \( t_{cr} \) as a fraction of the remaining life. It should be noted that the inspection intervals decrease with the decrease in the remaining life (the period from the inspection time \( t_i \) to the critical time, \( t_{cr} \)). This inspection strategy allows more inspections of the asset as it approaches the critical time to failure.

![Fig. 35: Inspection intervals as a fraction of the remaining life.](image)

6.2.2 Risk assessment

The probability of failure, \( P_f \), is defined as probability of non-detecting a growing flaw before reaching the critical time to failure, \( t_{cr} \). It is estimated as follows (Chung et al., 2006):

\[
P_f = \frac{\sum_{j=1}^{n} \prod_{i=1}^{n_j} (1 - POD(a_i))}{N_{sim}}
\]  

(79)

The cost of failure consequences is multiplied by the probability of failure to estimate the risk as follows:

\[
R_F = P_f \cdot k_F
\]  

(80)

where \( k_F \) is cost of failure consequences.

From Eqs. 79 and 80, the risk of failure is given by:

\[
R_f = \frac{\sum_{j=1}^{n} \prod_{i=1}^{n_j} (1 - POD(a_i))K_F}{N_{sim}}
\]  

(81)

6.2.3 Expected cost of inspections and repairs: The number of inspections between \( t_0 \) and \( t_{cr} \) is estimated based on the strategy shown in Figure 36. Since \( t_{cr} \) is a function of the material parameters, operating and environmental conditions, and the initial flaw size (\( a_0 \)). In general these are random variables. Monte Carlo simulation is used to account for all possible combinations of these random variables. The expected number of inspections, \( n \), is estimated as the average of number of inspections estimated in each simulation as follows:

\[
R_f = \frac{\sum_{j=1}^{n} \prod_{i=1}^{n_j} (1 - POD(a_i))K_F}{N_{sim}}
\]  

(81)
The expected cost of inspections, $C_I$, over the lifetime is estimated as follows:

$$C_I = n_s \cdot k_I$$  \hspace{1cm} (83)

where $n_s$ is sample size and $k_I$ is the cost of one inspection. When inspection and maintenance are planned for only one component; then, sample size, $n_s = 1$. However for a complete system or asset, it is often required to inspect a sample representative for the entire system/asset. For details of calculating inspection sample size for general and localized corrosion, see Khalifa et al. (2011a and 2011b).

From Eqs. 82 and 83, the expected cost of inspections is given by:

$$C_I = \frac{\sum_{j=1}^{N_{sim}} n_j \cdot n_s \cdot k_I}{N_{sim}}$$  \hspace{1cm} (84)

A repair will be undertaken if the detected crack is larger than the maximum acceptable crack size, $a_r$. Let $A(a_i) = \text{probability of acceptance of crack with size } a_i \text{ at the } i\text{th inspection } = P(a_i < a_r)$. The probability of repair at the $i\text{th}$ inspection is estimated as follows:

$$P_r(a_i) = POD(a_i) \cdot (1 - A(a_i))$$  \hspace{1cm} (85)

where $A(a_i) = 1$ if $a_i \leq a_r$ and $A(a_i) = 0$ if $a_i > a_r$.

The cost of repair, $c_r$, for each simulation of the flaw growth is estimated as follows:

$$c_r = \sum_{i=1}^{n_j} [P_r(a_i) \cdot n_s \cdot k_R]$$  \hspace{1cm} (86)

where $n_j$ is the number of inspections in the $j\text{th}$ simulation and $k_R$ is cost of one repair.

The expected cost of repairs, $C_R$, over the lifetime is estimated as the average repair cost from all simulations as follows:

$$C_R = \frac{\sum_{j=1}^{N_{sim}} c_r}{N_{sim}} = \frac{\sum_{j=1}^{N_{sim}} \sum_{i=1}^{n_j} [POD(a_i) \cdot (1-A(a_i)) \cdot n_s \cdot k_R]}{N_{sim}}$$  \hspace{1cm} (87)

where $N_{sim}$ is the number of simulations.

**6.2.4 Objective function:** The objective function, $OF$, is defined as the summation of the inspection cost, repair cost and risk of failure as follows:

$$OF = C_I + C_{Rep} + C_{Rep} + R_F - R_Y$$  \hspace{1cm} (88)
where \( C_{\text{Repl}} \) is the replacement cost. \( R_V \) is the remaining value of the asset or the system after a defined period \( T \) (study period).

Since the remaining life is different when using different maintenance options (repair or replacement), the remaining value after period of time, \( T \) is considered as a benefit for a maintenance option leading to a remaining life greater than \( T \). This is similar to what is carried out in cost/benefits analysis.

All costs in the objective function at different points of time are to be discounted to a present value when comparing different inspection and maintenance plans. The net present values, NPV of the cost of inspection, cost of repair and risk of failure are given as follows:

\[
NPV(C_i) = \frac{\sum_{i=1}^{t_i} \left[ \sum_{j=1}^{n_i} \frac{n_j k_j}{(1 + r)^j} \right]}{N_{\text{sim}}}
\]  
(89)

\[
NPV(C_x) = \frac{\sum_{i=1}^{t_i} \sum_{j=1}^{n_i} \left[ POD(a_j)(1 - A(a_j)) \frac{n_j k_x}{(1 + r)^j} \right]}{N_{\text{sim}}}
\]  
(90)

\[
NPV(R_x) = \frac{\sum_{i=1}^{t_i} \left[ \prod_{j=1}^{n_i} (1 - POD(a_j)) \frac{n_j k_f}{(1 + r)^j} \right]}{N_{\text{sim}}}
\]  
(100)

where \( r \) is the discount rate and \( t_i \) is time of the \( i \)th inspection.

The net present values of \( C_{\text{Repl}} \) and \( R_V \) are obtained as follows:

\[
NPV(C_{\text{Repl}}) = \sum_{t_{\text{Repl}}} \frac{C_{\text{Repl}}}{(1 + r)^{t_{\text{Repl}}}}
\]  
(101)

where \( t_{\text{Repl}} \) is time of replacement. It should be noticed that a maintenance option could include replacement many times during the lifetime.

\[
NPV(R_x) = \frac{R_x}{(1 + r)^t}
\]  
(102)

The net present value of the objective function, NPV(OF), can be estimated as:

\[
NPV(OF) = NPV(C_i) + NPV(C_{\text{Repl}}) + NPV(C_{\text{Repl}}) + NPV(R_x) - NPV(R_x)
\]  
(103)

The future annual costs of labour and materials tend to increase in future. An escalation rate may be used to predict inspection cost \( k_i \), repair cost \( k_r \) and failure cost \( k_f \) at different points of time.
6.2.5 **Optimization of RBIM plan:** The inspection intervals are selected as a fraction, R, of the remaining life (Figure 35). Figure 36 shows the cost (inspection and maintenance), risk of failure and the objective function for different values of the ratio, R. The costs of inspection and repair decrease as R increases because the smaller R the greater the expected number of inspections and repairs during the lifetime. As R increases, the risk of failure increases. The objective function is then estimated for different values of R as the sum of inspection cost (decreasing curve), repair cost (decreasing curve), replacement cost (constant), risk of failure (increasing curve) and remaining value (constant). The resultant objective function has a concave form (Figure 36). The optimum inspection interval is obtained using an R value determined by the minimum value of the objective function provided that the risk of failure does not exceed the pre-defined level.

![Graph showing Cost, Risk of Failure and Objective Function](image)

**Fig. 36:** Cost (inspection and maintenance), risk of failure and objective function versus different values of R for a suggested inspection and maintenance plan.

The solution of the optimization problem is repeated for different suggested inspection and maintenance plans and the minimum values of the objective function are compared to select the optimum inspection and maintenance plan.

For the application and more details of the developed approach for risk-based inspection and maintenance planning, see Khalifa et al., 2012 and Khan et al., 2012 appended to this report.

7. **Conclusion**

A framework for risk-based asset integrity management is developed. The framework consists of three main parts:

1. Development of a methodology for risk-based design
2. Development of a methodology for risk-based integrity modeling
3. Development of a methodology for risk-based inspection and maintenance planning.
In the first part, a methodology for risk-based design is introduced to calculate the burst pressure of corroded pipeline. The burst pressure and operating pressure are then used to develop a limit state equation. The failure probability of the pipeline was determined using different codes and standards; and models and comparisons were made. Fig 3 and Fig 7 depict the failure probability of internally and externally corroded pipeline determined for different codes. A scale of conservatism for codes and standards developed in Fig 4 and Fig 5 may be used for prior conception of conservatism of the codes and standards in the design if the design scenario is based within the bound of 0.15≤d/t≤0.42. The validation of failure probability, calculated by FOSM with experimental tests, is difficult to obtain since it requires significant test data to get a standard deviation of burst pressure. However, Monte Carlo simulation verifies the results obtained by the FOSM method. As the result obtained by two methods closely match, the accuracy of the result is ensured. This paper further investigated the cause of variability in probability of failure that must be minimized for risk-based design approach. It is observed that different ways of accounting the defect geometry in the burst models are responsible for variation in estimated remaining strength. This variation is highly responsible for significant variability in probability of failure. It is therefore recommended that modelers or classification societies concentrate on reduction factor, Pbi for a unified risk-based design approach. This will provide an identical probability of failure (Pf) of the burst models recommended by codes/standards.

In the second part, a methodology for risk-based integrity modeling is developed. This methodology is capable of making optimal maintenance and replacement decisions for process components. Structural degradations are random processes and thus, probabilistic models are developed to accurately predict failure mechanisms. The life threatening component degradation processes are identified as different types of corrosion and cracking. The degradation processes include UC, PC, EC, SCC, CFC and HIC. These structural degradations are modeled using prior distributions, which are subsequently updated using NDT data to posterior distributions through the use of Bayes’ theorem. The simulation based M-H algorithm and analytical Laplace approximation methods are used to develop the posteriors. Since these posterior models are based on real life NDT data, they provide more reliable and accurate predictions for the future degradations of components. The development of an RBIM framework using the potential degradation mechanisms was discussed. The prior distributions for various degradation processes are developed based on the data extracted from literature. The relative accuracy of the prior models are tested using probability plots and A-D tests, and the parameters are estimated using the methods of least square and maximum likelihood estimates. The model was applied to a real life case study, using field NDT data from an ageing offshore process facility. Literature data is used for estimating the likelihoods of cracking. The posterior probability models are then developed. The use of a simulation method is necessitated because none of the prior-likelihood models falls into the natural conjugate pairs of the exponential family. Two MATLAB codes, one using the M-H algorithm and the other using the Laplace approximations, have been developed and used to compute the posterior distributions. These codes are calibrated using known conjugate pair estimates. The MATLAB codes performed well for Weibull, Lognormal and Type 1 Extreme Value distributions with two and three parameters. The posterior probability thus developed is useful in assessing the potential risk to the life of component. Further, it has been observed that the rejection sampling based M-H algorithm is the more suitable method compared with the Laplace approximation for posterior estimation of components. Using the M-H algorithm, it is observed that the posterior probability model that can be used to estimate the future failure probability due to the UC is 3P Weibull; the PC is Type1 Extreme Value, and the EC is by 3P Weibull. Similarly, the SCC degradation can be best modeled by Weibull; the CFC and HIC by Lognormal and Weibull distributions. An economic consequence analysis model based on the component’s minimal repair and replacement concept is discussed. The consequences of failure are estimated by developing the cost of failure, inspection and maintenance. The cost of failure includes the loss due to breakdown, loss due to shutdown, the cost of a cleanup strategy, loss of nature damage and liability. The cost of inspection and maintenance are estimated from their unit costs as obtained from an inspection and maintenance company. Then, the CDF of posterior probability and AEC are combined to
produce the operational risk curve. The optimal inspection and maintenance interval is determined from the operational risk curve at the point corresponding to the minimum risk. In this article, the optimum inspection and maintenance interval is observed to vary from 4 to 6 years for different corrosion and cracking processes. The smaller value (4 years) should be considered the optimum maintenance interval. This interval should be revised as new NDT data is obtained. The developed model may be applied to the optimization of inspection and maintenance even though component degradations follow non-conjugate pairs. This model could be refined further by incorporating the actual costs and rates of interest based on the market value at the time of analysis. An economic consequence analysis based on component replacement concept is discussed. The replacement strategy is based on the economic service life of the component and the threat from an imminent failure of component. The replacement intervals based on critical degradation processes are presented in Table 17. The smallest one is considered. The smallest one is 7 years for SCC and HIC degradations, which is reported as the optimum replacement interval. By performing replacement at this interval, the risk will be reduced to ALARP level and replacement intervals will be maximized.

In the last part, a methodology for risk-based inspection and maintenance planning is introduced. This methodology provides:

- Bayesian sample size calculation for inspection of general corrosion.
- Sample size estimate to assess localized corrosion.
- Sample size formula for the extreme value analysis of localized corrosion in two cases: with having prior information and without having prior information.
- A quantitative risk-based inspection and maintenance (RBIM) planning. This comprises of main steps: classification of asset’s components/areas according to criticality of deterioration, asset deterioration modeling, risk assessment, cost estimation (inspection and maintenance) and optimal selection of inspection interval and maintenance strategy.

8. Tools developed

To implement the above methodologies and models, Matlab software is used for effective application of these methodologies and models.

9. Publications


6- Khalifa M., Khan F., and Haddara M. (2012). Quantitative risk-based inspection and maintenance for process components. Submitted for publication in the 25th International Congress on Condition Monitoring and Diagnostic Engineering Management (COMADEM), which is being held in Huddersfield, UK on 18th - 20th June 2012.


10.1 Future work: risk-based integrity modeling

An attempt has been made to develop a risk based integrity modeling for the optimal maintenance decision making of offshore process components. This work can be extended as suggested below:

a) Non-age dependent degradation processes modeling: This study has been limited to the age-dependent degradation processes of process components. Other non-age dependent failure mechanisms such as third party damage; ship or boat collision; material and fabrication defects; operational errors; vibration and cyclic stresses may be investigated further.

b) The online risk monitoring systems: This study is limited to dynamic updating of the system performance in terms of revised inspection and maintenance strategy. However, if the risk can be monitored online, the system performance can be tracked and the maintenance decisions may be taken on the spot. Such a system will be versatile considering the imminence of failures. There is a broad scope for such studies especially for the far, deeper offshore facilities.

c) System effects in the risk analysis: This study has been focused on the component level risk analysis. However, the risk analysis shall be conducted on a system level. The approach developed in this thesis can be extended and applied to a group of components which constitute a system. The system safety analysis may be achieved through the fault tree and event tree analysis.

d) Risk analysis for combined degradation mechanisms: In reality the degradation process occurs simultaneously, for e.g., UC and SCC, PC and CFC, etc. In this study, they are assumed to be independent and isolated. The modeling of coupled degradation process is a challenging task, which needs to be explored further.

10.2 Future work: risk-based inspection and maintenance planning

a) Inspection sampling: The classical concept in inspection sampling is to ensure that the deviation between the sample estimate and the population estimate (sampling error) is within an acceptable error. In future, a methodology for risk-based sampling (RBS) will be introduced. This methodology aims to estimate the required sample size to assess deteriorated assets due different damage mechanisms based on a non-classical concept. This concept is that the estimated sample
size should ensure with a specified confidence level that the risk of failure of both inspected and non inspected components/areas does not exceed a pre-defined limit.

b) Failure modeling: Physical failure modeling is used in the introduced methodology for risk-based inspection and maintenance planning. This methodology is useful for fixed equipment such as pipelines, vessels, heaters towers and heat exchangers subjected to damage mechanisms such as fatigue and corrosion. For rotary equipment such as pumps, compressors and turbines, failure occurs randomly due to different reasons such as mechanical seal failure, bearing failure, shaft misalignment and incorrect installation. Statistical failure modeling is better used for this kind of failure in rotary equipment. The new methodology would integrate the statistical failure modeling in a model for minimizing the life cost including inspection and maintenance cost without compromising the safety. It would also be used to make a decision to continue the operation of aging rotary equipment or to replace it with new equipment. The optimal decision is made based on a balance between the life cost and risk of failure.

References


Khalifa M., Khan F., and Haddara M. (2012). Quantitative risk-based inspection and maintenance for process components. Submitted for publication in the 25th International Congress on Condition Monitoring and Diagnostic Engineering Management (COMADEM), which is being held in Huddersfield, UK on 18th - 20th June 2012.


**Appendices**

The publications listed in Section 8 are appended to this report.